1949

APPORTIONMENT OF THE HOUSE OF REPRESENTATIVES

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IN 1950 a census will be made of the United States. On the basis of that census the House of Representatives must be reapportioned, in order that each state may have the number of seats in the House to which the latest figures on its population entitle it. The criterion by which this reapportionment is to be made is set out by Article I, Section 2 of the Constitution, as amended by the Fourteenth Amendment:

"Representatives shall be apportioned among the several States according to their respective numbers, counting the whole number of persons in each State, excluding Indians not taxed. . . . The number of representatives shall not exceed one for every thirty thousand, but each State shall have at least one representative."

It might seem that apportioning the House according to these directions is as easy as giving two senators to every state. That it is not so simple is shown by the controversy which has occurred every decade since the founding of the Republic over the fairest method of implementing the constitutional requirements.¹

When the apportionment is made after the 1950 census, it will be made by one of five methods which have been developed for that purpose. The issue has already been reopened as to which of these methods comes closest to meeting the constitutional requirements.² But this is not the normal kind of constitutional issue which can be answered by looking to the words of the Constitution; the real controversy concerns selection of the method of apportionment closest to the standard, which the Constitution must have...
implied, of being fair to every state. To resolve that controversy, mathematics must come to the aid of law.

Three things must be determined in apportioning the House: the proportion of the national population which resides in each state; the total size of the House; and the number of representatives to be allotted to each state in order that its proportion of the total membership of the House may most closely approximate its proportion of the total population. The first of these determinations is not a mere matter of counting noses, but the Bureau of the Census is now so adept at its task that it can be expected to report correctly the population of each state. Interest today centers, therefore, on the more difficult problems of deciding on the size of the House and equitably allocating that number of representatives to the several states.

subject of apportionment for the last forty years, has already announced his support of proposals to reduce the size of the House each decade and to make the apportionment by a method different from that now provided for by law, for which he finds more support in the Constitution. Willcox, Letter to the Editor, N. Y. Times, Jan. 9, 1949, § 5, p. 8, col. 5. See note 32 infra.

Prof. Willcox has always maintained that the problem of apportionment is wholly a political problem, rather than a mathematical one, and his conclusions have not always been in accord with the bulk of thinking in this field. Since he is not a mathematician he seems to have been guilty of occasional unimportant errors concerning technical matters during the give and take of testimony before Congressional committees. Other writers have found it good sport to collect Willcox’s lapses linguae into chapters entitled “Errata in the Current Literature,” Huntington, Methods of Apportionment in Congress 37–40 (Sen. Doc. No. 304, 76th Cong., 3d Sess. 1940); cf. Schmeckebier, Congressional Apportionment 69–8 (1941); or to scatter corrections of such errors in pontifical footnotes throughout their work, see, e.g., Chafee, Congressional Redapportionment, 42 Harv. L. Rev. 1015 (1929) passim. Such criticism should not diminish Prof. Willcox’s stature as a thinker who has attacked the problems of Congressional apportionment with great originality, forcefulness, and devotion.

3. Proposals are frequently advanced to exclude aliens from the enumeration of the population for apportionment purposes, or to base apportionments upon the number of votes cast in the last Presidential election, and thus effectuate the provision of the Fourteenth Amendment that a state’s representation be decreased for any persons the state disenfranchises. Although persuasive arguments have been advanced on behalf of such proposals, Schmeckebier, op. cit. supra note 2, at 85-105, it is unnecessary to consider their merits. The easy answer is that neither proposal is constitutional, as Schmeckebier admits, and a constitutional amendment along such lines would have no chance of ratification.

4. It has not always been so. “In 1870 there were many odd errors made in certain classes of statistics. There were reported 131 out of 151 colored children in one family as insane in a city in Massachusetts; infants died of delirium tremens, and old men of teething, and people were frozen to death South in August, and sunstruck North in January.” 11 Cong. Rec. App. 99 (1881). And even in this century, it has been an unimaginative Congressman indeed who was unable to give good reasons why census figures unfavorable to his state should not be relied upon. The statement of Representative Rhodes, of Missouri, is typical: “... I wish to protest against the passage of such legislation as will reduce the membership of my State in this body; and I will tell you why. At the time the census was taken last year an unusual industrial condition prevailed in Missouri. Thousands and tens of thousands of our working people at that time were
Although it is commonly thought that the size of the House is permanently fixed at 435, Congress has the power to increase or decrease the number of representatives to any figure it may choose, and history indicates that Congress is not likely to be hesitant about altering the size of the lower chamber. Until 1929 no two apportionment acts had ever provided for the same size House. In 1850 Congress enacted a measure purporting to limit the size for all time to 233, but this limitation has been consistently ignored.

The present size of the House was arrived at purely by chance. The apportionment in 1911 provided for a House of 433 members since that was

<table>
<thead>
<tr>
<th>Year</th>
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<tr>
<td>1790</td>
<td>106</td>
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<td>1800</td>
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<td>1910</td>
<td>435</td>
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The Hon. John E. Rankin, of Mississippi, complained that the census failed to show the true population of his state because "[D]uring the war and just before the war, when labor was scarce, there was a great song that went throughout our section among the Negro population to the effect that they could get better wages elsewhere, and that was a fact at the time, and a great many of them left the State on that account. For the last 18 months these negroes have been pouring back into Mississippi and begging the landlords to take them back.... They took this census in January, and in some instances they appointed some old rundown politicians to supervise the work, and as a result of this inefficient census taking a great part of the agricultural population of my State will be deprived of representation on the floor of this House." Id. at 65. Representative Rankin also put much stress on the fact that the census was made during the winter, when Mississippi has an excessive amount of rainfall, instead of in May and June. Ibid.

5. The Constitution makes no provision as to the size of the House except to direct that there shall not be more than one representative to each thirty thousand inhabitants, and that each state shall have at least one representative. U. S. Const. Art I, § 2.

6. Apportionment acts after each census have provided for a House of the following sizes:

7. 9 Stat. 432 (1850).
8. 37 Stat. 13 (1911).
the lowest number which would prevent any state from losing a representative. 9 The admission to statehood one year later of Arizona and New Mexico forced the size of the House to its present level without any thought that a House of 435 members would be permanent. 10 After the census of 1920 the House Committee on the Census reported a bill fixing the size of the House at 483, the smallest size which would cause no state to lose a representative. The House amended this to provide a body of 435 members, but the Senate killed the bill because many states would have lost seats. A new measure was then reported which would have apportioned 460 members, causing two states to lose one representative each. By a margin of four votes, the House defeated this bill. In 1927 another bill which would have provided a House of 435 members was defeated. 11

In order to prevent the recurrence of a decade without an apportionment, Congress passed a Permanent Apportionment Act in 1929. 12 This provided that after each census the Secretary of the Interior should submit a table showing the number of representatives to be allotted to each state under the two most important methods of apportionment, equal proportions and major fractions. If Congress failed to act within sixty days, the allotment based on the method used in the last previous apportionment was to go into effect automatically. It was necessary in the act to specify for what size House the Secretary of the Interior should prepare his tables, and Congress wrote into law the figure 435. The Permanent Apportionment Act was amended in 1941 13 to require use of the method of equal proportions, but the size of the House was not changed.

The present “fixed” limit on the size of the House means that some states will have their number of representatives reduced at any apportionment. 14 The difficulties which this causes should not be underestimated. The legislatures of the states in question must choose between reapportioning their states or, by default, permitting their entire Congressional delegations to be elected at large. 15 The apportionment acts from 1842 to 1911, which

10. The committee report on the 1911 act opposed any attempt to limit permanently the size of the House, and expressed doubt as to the constitutionality of making apportionments on any other than an ad hoc basis. Id. at 5.
11. SCHMECKEBER, op. cit. supra note 2, at 120-1, summarizes this and gives citations of the unsuccessful bills.
14. The sweeping population movements of this decade will make this a particularly irksome problem in the apportionment after the 1950 census. Apportionment by each of the five possible methods according to July, 1948, population estimates shows that in a House of 435 members, the delegations of eleven states would be reduced by two methods, the delegations of twelve states by two methods, and the delegations of seventeen states by one method. See Appendix B, infra, p. 1386.
invariably required that representatives be elected from districts, are evi-
dence that sound public policy is opposed to electing Congressional del-
egations at large.16 But legislatures attempting to conform to the public
policy of electing Congressmen by districts are forced by political expedi-
tency to gerrymander, and gerrymandering has the effect of making repre-
sentative government less representative by shaping districts to sterilize
as many minority votes as possible.

The classic arguments in favor of limiting the size of the House are that a
larger House would be administratively unwieldy, and that it would more
likely be guided by passion than by reason. "Had every Athenian citizen
been a Socrates, every Athenian assembly would still have been a mob." 17
But such argument presupposes that the House is still a deliberative body
and that its most important function is its floor debate. Every page of the
Congressional Record testifies eloquently that this is not the case. The rules
limiting debate and the great power wielded by the leadership reduce floor
proceedings to a few meaningless speeches for the sake of the record. The
important function of the House today is that of a huge voting machine
registering the temper of the populace.

And a larger House would be a more precise instrument for indicating this
public sentiment. Today each Congressman represents—in theory—301,000
persons. Clearly it would be impossible to select 301,000 persons with such
similar views that they could be adequately represented by one person.
When it is considered that the normal Congressional district does not con-
sist of persons chosen for their homogeneous views, but rather of persons
happening to live in a particular area, it is apparent how unrepresentative
government can be. Congressional districts of 30,000, the constitutional
minimum, would not be a complete cure for this situation, but they would
go a long way toward eliminating districts in which silk stocking areas are
combined with slums.

Other advantages inhere in an increase of the size of the House. The larger
the House, the more exactly each state's proportion of the total membership
can be made to coincide with its proportion of the total population of the
nation. And the smaller the size of the average Congressional district, the
more difficult it becomes for a state legislature to gerrymander the state.
A large House would obviate the dilemma presently confronting the lower
chamber of having either to cut down the number of committees—and so
increase the amount and complexity of the work within each committee—
or to require each Congressman to serve on so many committees that he is
unable to do a thorough job on any of them.18 A large House would mate-

16. The requirement that representatives be elected from districts has never been
enforced. See 1 HINDS, PRECEDENTS OF THE HOUSE OF REPRESENTATIVES 170-2 (1907).
18. The Congressional Reorganization Act of 1946, 60 STAT. 812 (1946), reduced the
number of standing committees in the House from forty-eight to nineteen, in an effort to
reduce the number of committee assignments of each representative. But the complexity
rially increase the number of representatives with special competences to give expert consideration to the wide variety of subjects which now affect the national interest, while smaller constituencies would enable each Congressmen to give closer attention to the individual needs of the citizens he represents. And finally it just makes sense that as the government and the nation grow bigger, so should the House of Representatives grow bigger.

These advantages of a large House could be obtained by one drastic increase in the membership of that body, but it seems more in accord with the democratic tradition to build up the size of the House by smaller but frequent increases. Such a plan would permit the House to adapt its methods and techniques to a larger membership in a more gradual manner. The simple way to provide for such increases is to amend the Permanent Apportionment Act to provide that after each census the Secretary of the Interior prepare a new apportionment on the basis of the smallest size House in which no state would lose a member, rather than on the basis of a House of 435 members.

PROBLEMS OF ALLOTING REPRESENTATIVES TO STATES

After the size of the House has been determined and the population of each state and of the nation is known, the most difficult problem in apportionment is still to be met—the allocation of representatives to states. The work is such that it is impossible for a small number of committees to do a satisfactory job. Three months after the Eightieth Congress had been organized—the first Congress to be organized in the new "streamlined" manner—a superstructure of two special committees and 119 subcommittees had been erected above the nineteen committees into which the Reorganization Act had divided the House. N. Y. Times, March 20, 1947, p. 23, col. 1.

19. A senatorial secretary is quoted as saying: "Today senators are just messenger boys. Remember, the Federal Government has something to say about the house you live in, your wages, fuel, food and clothing prices, the raw material for your factory— all this in addition to the usual Federal activities. No wonder the citizens need help. And even when it's something that doesn't concern Washington, they write anyway." White, Anything for a Constituent, Saturday Evening Post, Oct. 26, 1946, p. 30, 59. Representatives, generally even better known to the voters than are Senators, are likely to be even more burdened with constituents' requests.

20. On the basis of July, 1948, population estimates, a House of 490 members would be required in order for no state to lose a seat if the method of equal proportions, for example, were used in making the apportionment. In a House of 461 members, only Arkansas would lose a seat, while all other states remained unchanged or gained. See Appendix B, infra, p. 1386.

Estimates of maximum population range from "perhaps 165 or 170 million" to be reached about 1980, Ogburn, Who Will Be Who in 1980, N.Y. Times Magazine, May 30, 1948, p. 23, 34, to "about 196,631,000" to come "well after the year 2000," Prof. Raymond Pearl, quoted in Potter, The Story Behind the Story, Esquire, March, 1949, p. 40. Since the Constitution limits the size of the House to one representative to every thirty thousand inhabitants, the maximum size of the House under the rule contained in the text would be 5100 members by the lower population estimate, or 6556 members by the higher estimate. But the size of the House would not reach such a figure until several centuries after the population had reached its maximum.
Instinctively the proper method would seem to be to divide the number of representatives into the population of the nation to find out how many people there should be to each representative. This figure would then be divided into the population of each state, and the result would be the number of representatives to which the state is entitled. The trouble is the extreme unlikelihood that a state's quota will ever be an exact whole number, and it is somewhat difficult to elect a fraction of a Congressman. Yet one state may deserve 3.01 representatives, and another 3.99. Are they each to be given three? Or each four? Or should the first state be given three and the second four representatives? Congress has had much difficulty in answering such questions so as to produce the least inequity, and only within this century has there been any valid mathematical analysis of the problem.

Terminology

In order to simplify the discussion of possible methods of apportionment, it is desirable first to define certain of the terms most often used.\(^2\)

The *ratio* is the figure obtained by dividing the number of representatives to be apportioned into the total population of the nation. Thus the 1940 census listed a population of 131,006,184; 435 representatives were to be apportioned. The ratio, therefore, is 131,006,184 divided by 435, or 301,164. Modern methods of apportionment make no use of the ratio. While it is possible to derive a figure from them, these artificial ratios play no part in the apportionment process, and are of interest only to Congressmen unable to follow the intricacies of the mathematical methods.\(^2\)

The *quota* is the number of representatives which a state is awarded under a method of apportionment, to be differentiated from the *exact quota*, which is the number of representatives to which the state is entitled. Under early methods of apportionment the ratio was divided into the population of a state to determine its exact quota. If the ratio was 300,000 a state with a population of 725,000 would have an exact quota of 2.42. The quota which would be allotted it would be either 2 or 3, depending on the method of apportionment being used.

Where the exact quota consists of a whole number and a fraction, the fraction is classified as a *major fraction* if it is equal to or greater than one-half, and *minor fraction* if it is less than one-half.

The *average district* is the average population per district in a particular state. It is determined by dividing the population of the state by the quota which a particular apportionment assigns the state, so that it is also the

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\(^2\) The terminology employed here follows generally that of [Schmeckebier](#), *op. cit.* supra note 2, since that text, a 233-page study for the Brookings Institution, bids fair to be the definitive work on Congressional apportionment.

\(^2\) Under the method of apportionment used after the 1940 census, any figure from 300,473 to 300,796 could have been assumed as the "ratio." 300,635 was the figure most used in Congressional debates. As is shown in the text, the natural ratio was 301,164.
average population per representative. The apportionment of 1940 gave
Connecticut, for example, a quota of six representatives for its population
of 1,710,112. Dividing the population by six, the average district is found
to be 285,019.

The individual share in a representative, or, more briefly, the share, is
derived by dividing the quota for a state by the population of the state, so
that mathematically it is the reciprocal of the average district. Using the
1940 figures, the share for a Connecticut individual was six divided by
1,710,112, or .00000351. For the sake of simplicity, it is customary to
multiply this figure by one million, and speak of share per million of pop-
ulation. For Connecticut this would be 3.51 per million.

The absolute difference between two numbers is determined by subtracting
the smaller from the larger. The relative difference is the percentage by
which the larger exceeds the smaller. The absolute difference between a
share of 10.00 and a share of 11.00 is 1.00, exactly the same as the absolute
difference between a share of 5.00 and a share of 6.00. But the relative
difference in the first case is the absolute difference of 1.00 divided by 10.00,
or 10%, while the relative difference in the second case is 1.00 divided by
5.00, or 20%.

Each of the modern methods of apportioning the House uses a priority
list which shows the quota to which each state is entitled for any particular
size of the House. As will be seen later, each mathematical formula for
apportionment ends up by giving a list of fractions known as multipliers.23
The population of each state is multiplied by each multiplier taken from
tables prepared for that particular method. This gives for each state a list
of numbers of decreasing magnitude, known as rank indices. The priority
list is then constructed by first listing each state once to comply with the
Congressional guarantee that each state shall have at least one representa-
tive. All the rank indices are then arranged in a single series in descending
order of size, so that the largest rank index indicates which state should be
awarded the forty-ninth representative, the next largest the fiftieth, and
so on. For any particular size of the House all that is necessary is to count
the number of times a state appears on the priority list up to that size of the
House to see how many representatives the state should have in a House
of the specified size. An example of the construction of a priority list is
given in an appendix.24

**Paradoxes**

Any method of apportionment which involves a fixed ratio is subject to
the population paradox, in which an increase in the total population may

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23. As is seen in the development of the modern methods of apportionment, p. 1371
*infra*, the mathematical processes from which each method is derived yield a series of
divisors to be successively divided into the population of a state. It is purely as a matter of
convenience that the reciprocals of these divisors are taken and used as multipliers.

result in a decrease in the size of the House. The classic hypothetical example shows how it would be possible for the size of the House to drop from 435 to 391 while the population of the United States is increasing from 102,750,113 to 102,958,798 if a fixed ratio of 250,000 were being employed.

The theory of the example is that one state which originally had just a few more persons than it needed to be entitled to a certain quota might increase its population so that it would have just a few less people than it needed for an additional representative. Its population would have increased almost 250,000 but it would get no larger quota. But another state which also started out with just a few more people than it needed for a certain quota would have to lose only a very few people to fall below the dividing line and lose a representative. The combined population of the two states is increased almost 250,000 but their combined number of representatives is decreased by one.

The Alabama paradox to which some apportionment methods are subject was first discovered in 1881 when tables were prepared showing the apportionment for a House of several different sizes. In a House of 298 members, Alabama's quota was seven. If the House were increased to 299, Alabama would get the additional representative, so that its quota would be eight. But mirabile dicu, in a House of 300 members Alabama's quota would shrink to seven again. And in a House of 301, Alabama would once again get an eighth seat, this time permanently. Since that time, any situation in which an increase in the size of the House decreases the size of the delegation from a particular state has been known as the Alabama paradox.

Such paradoxes as these were the curse of early haphazard apportionment methods. They cannot occur in the modern methods which are the product of more incisive mathematical analysis.

Outmoded Methods of Apportionment

Before considering the modern mathematical methods of apportionment, it is instructive to consider those methods which have been used and discarded or proposed and rejected heretofore.

25. See Chafee, supra note 2, at 1025-6.
26. Full details of this example may be found in Huntington, op. cit. supra note 2, at 18-19.
27. For an explanation of the Alabama paradox and why it occurs, see the letter from General F. A. Walker to Hon. S. S. Cox, printed in Apportionment Under Tenth Census 19 (1881). An extreme example of the Alabama paradox was discovered in a proposed apportionment after the census of 1900 when Maine had three seats in a House of 382, four seats in a House of 383, 384, or 385, three in a House of 386, four out of 387 or 388, three in a House of 389 or 390, and four in a House of 391. "Now you see it and now you don't," commented Representative Littlefield of Maine, "In Maine comes and out she goes. The House increases in size and still she is out. It increases a little more in size, and then, forsooth, in she comes. A further increase, and out she goes, and then a little further increase and in she comes. God help the State of Maine when mathe-
Method of Rejected Fractions

The first five apportionments, up to 1830, were made by a method devised by Thomas Jefferson known as the method of rejected fractions. In this method the natural ratio was found and the exact quota for each state computed, and the states were assigned a quota equal to the whole number part of the exact quota, with any fractional part of the exact quota rejected. By this method a state with an exact quota of 3.99 and another with an exact quota of 3.01 would each be given three seats in the House. In addition to this manifest inequity, this method is subject to the population paradox, in which an increase in the population may reduce the size of the House.

Method of Included Fractions

A companion method to rejected fractions is the method of included fractions, in which the exact quotas are computed and each state given the next highest whole number of representatives, so that the states with exact quotas of 3.01 and 3.99 would each be assigned four seats. This method is also manifestly inequitable, it is subject to the population paradox, and has never been used in an apportionment.

Method of 1840

The apportionment after the 1840 census was made according to a method devised by Daniel Webster which gave a state a seat for every whole number in its exact quota and an additional seat if the fractional part of the quota exceeded one-half. By this method the state with an exact quota of 3.01 would be assigned three seats, while the state with the exact quota of 3.99 would be assigned four seats. This method is subject to the population paradox, and further it is impossible to determine in advance what the size of the House will be.

Vinton Method

Apportionments from 1850 to 1900 were made by the Vinton method, named after the Congressman who authored it. This method assigned to each state a quota equal to the whole number part of its exact quota, and awarded the remaining seats necessary to fill out the House to the states with the largest fractions in their exact quotas. This is the method which was being used when the Alabama paradox first appeared. A suggested improvement was devised, known as the modified Vinton method, which would have assigned seats for whole numbers and given the remaining seats...
to the states whose fractional quotas divided by their populations were largest. This modification is still subject to the Alabama paradox.

**Method of Geometric Fractions**

The method of geometric fractions would have given each state a representative for each whole number in its exact quota and one additional representative if the exact quota was greater than the geometric mean of the number of seats already given the state and the number of seats already given plus one. For example, a state with an exact quota of 1.40 would be given one seat and a state with an exact quota of 1.42 would be given two seats, since the geometric mean of one and two is the square root of one multiplied by two, or 1.414. This method is subject to the population paradox, and has never been used in an apportionment.

**Method of Harmonic Fractions**

The method of harmonic fractions would have given a state an extra seat for its fraction if the exact quota of the state exceeded the harmonic mean of the number of seats already assigned the state and that number plus one. The harmonic mean of two numbers is twice their product divided by their sum, so that a state with an exact quota of 1.32 would be given one representative while a state with a quota of 1.35 would be assigned two seats, since the harmonic mean of one and two is 1.33. This method was proposed by Prof. James Dean in 1832 as an appendix to a famous report on apportionment by Daniel Webster. It is subject to the population paradox, and has never been used in an apportionment.

**Minimum Range; Inverse Minimum Range**

Minimum range and inverse minimum range are not properly apportionment methods, but rather are tests by which to measure the success of an apportionment. The test of minimum range says that an apportionment is satisfactory when the difference between the largest and the smallest average district has been reduced to a minimum; the test of inverse minimum range seeks to minimize the difference between the largest and smallest individual share. Apportionments satisfactory by this method may contain the Alabama paradox, and the tests are no longer in use.

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32. But Willcox, *supra* note 2, apparently is urging a process based on the test of minimum range: "The decisive question for Congress is: Does an apportionment under the proposed method give results which come nearer to meeting the requirement of the Constitution than those reached by the present or any other method? The unattainable ideal is so to apportion seats as to equalize the district population of the states. . . . The measure of nearness to this ideal which seems the best and simplest and most likely to appeal to Congress is the difference between the largest and the smallest district population."
Method of Alternate Ratios

Although the method of alternate ratios, devised by Dr. Joseph A. Hill, chief statistician of the Bureau of the Census, was never used for an apportionment, it was the first sophisticated method to be devised, and pointed the way to modern mathematical methods of apportionment.\(^\text{33}\) This method gave a seat for each whole number in the exact quota. A priority list was then prepared by computing for each state the geometric mean of the average district if an additional representative be given the state and the average district if an additional representative be denied. Representatives could be given to states in the order in which they appeared on the priority list until the House reached any predetermined size. This method is subject to the Alabama paradox.

MODERN METHODS OF APPORTIONMENT

In 1832 a committee under the chairmanship of Daniel Webster reported to the Senate in part:

"The constitution, therefore, must be understood not as enjoining an absolute relative equality—because that would be demanding an impossibility—but as requiring of Congress to make the apportionment of representatives among the several States according to their respective numbers, \textit{as near as may be}. That which cannot be done perfectly, must be done in a manner as near perfection as can be. If exactness cannot, from the nature of things, be attained, then the greatest practicable approach to exactness ought to be made." \(^\text{34}\)

This classic statement is the key to present methods which are based on making the quota a state is given "as near as may be" to the exact quota to which the state mathematically is entitled.

\(^{33}\) Dr. Hill's statement of his method is found at H. R. Rep. No. 12, 62d Cong., 1st Sess. 43 (1911). In submitting the method he said: "It was not my expectation that this method would be applied in the pending apportionment." \textit{Ibid.} And in a letter to a scholar who endorsed his method, he said: "I did not think it advisable at this time to urge the adoption of my method or agitate for it, especially in view of the fact that, as applied to the existing states, it gives exactly the same result in the apportionment of 433 representatives as the method endorsed by Professor Willcox and favored by the Committee. The only difference would be that my method would bring in the territory of New Mexico, when it becomes a state, with two representatives instead of one. I was not disposed to hold a brief for New Mexico or make that territory my client; I think one representative is quite enough for that state. At any rate I am satisfied that the Committee would not have endorsed the method at the present time, even if I had brought to bear all the pressure at my command." Letter to Prof. Irving Fisher from Dr. Joseph A. Hill, dated Oct. 5, 1911, in the Yale University Library. But the method of alternate ratios was found to be subject to the Alabama paradox, and by 1927 Dr. Hill could say, "I have no method of my own; the method of equal proportions has superseded my method." \textit{Huntington}, \textit{op. cit. supra} note 2, at 29.

\(^{34}\) \textit{Report of Select Committee on Representation}, Sen. Doc. No. 119, 22d Cong., 1st Sess. 4 (1832) (italics are those of the original author).
Fundamental Theorem of Apportionment

If the populations of two states are $A$ and $B$, and the quotas assigned to them are $a$ and $b$, in an ideal apportionment $A:B::a:b$. This proportion can be expressed in the form of an equation in four ways: $A/a = B/b$; or $a/A = b/B$; or $A/B = a/b$; or $B/A = b/a$. In practice it is impossible to satisfy any of these equations exactly, but it may be set out as the Fundamental Theorem of Congressional apportionment that in each of these equations the left side of the equation should be made "as near as may be" equal to the right side of the equation.

These four equations may be expressed verbally as follows: the average district in each state should be as nearly equal as possible ($A/a = B/b$); the individual share in each state should be as nearly equal as possible ($a/A = b/B$); if state A is twice as populous as state B, it should have twice as many representatives, as nearly as may be ($A/B = a/b$); if state B is half as populous as state A, it should have half as many representatives, as nearly as may be ($B/A = b/a$). Only one further postulate need be assumed in order to derive from each of these equations a mathematical method of making an apportionment so as to satisfy the particular equation from which the method is constructed: in a satisfactory apportionment it should be impossible to make one of the above equations any more nearly equal by a transfer of a representative from one state to another.

Method of Major Fractions

Of the four methods which may be simply derived from the equations of the Fundamental Theorem, only the method of major fractions has ever been used in an actual apportionment. This method takes the equation $a/A = b/B$ and proposes to reduce to a minimum the absolute difference between the individual shares in any two states. This difference is expressed in the form $a/A - b/B$, and is used as a test to decide whether it would be fairer to assign an additional representative to state A rather than to state B. If the additional representative is assigned to state A, the absolute difference of the individual shares would be: $(a + 1)/A - b/B$. If the additional representative be given to state B, the absolute difference would be $(b + 1)/B - a/A$.

If the assignment of the additional representative to A is correct, the absolute difference will be smaller than if the additional representative had been assigned to B, or:

$$\frac{a + 1}{A} - \frac{b}{B} < \frac{b + 1}{B} - \frac{a}{A}.$$  

Transposing terms:

$$\frac{a + 1}{A} + \frac{a}{A} < \frac{b + 1}{B} + \frac{b}{B}$$

and grouping:

$$\frac{2a + 1}{A} < \frac{2b + 1}{B}.$$
Inverting the inequality, this becomes:
\[
\frac{A}{2a + 1} > \frac{B}{2b + 1}
\]
and multiplying each side of the inequality by two:
\[
\frac{A}{a + \frac{1}{2}} > \frac{B}{b + \frac{1}{2}}.
\]
This inequality means that it will be fairer to assign an additional representative to state A than to state B if the population of A divided by its present number of representatives plus one-half is greater than the population of state B divided by its present quota plus one-half.

From the test just found it is possible to construct a priority list by which to apportion the House according to the method of major fractions. After each state has been given the one representative the Constitution requires, the next seat should be given to the state whose population divided by 1.5 is greatest. The same state should be given the fiftieth seat if its population divided by 2.5 is greater than that of any other state divided by 1.5. And so it goes: the population of each state is divided successively by 1.5, 2.5, 3.5, . . . , and the rank indices thus obtained are arranged in a priority list. The apportionment made from that priority list will have a smaller absolute difference between the individual shares of any two states than would any other apportionment.

The method of major fractions was devised by Prof. Walter F. Willcox, of Cornell University, in 1910, and was used in the apportionments of 1911 and 1931.\(^{35}\) It was one of the two methods provided for by the Permanent Apportionment Act of 1929, but the act has since been amended to eliminate this method. As will be seen, the method of major fractions tends to favor slightly the larger states.\(^{35}\)

**Method of the Harmonic Mean**

The method of the harmonic mean uses the equation \(A/a = B/b\) and proposes to minimize the absolute difference between the average districts in any two states. By manipulation similar to that demonstrated for the method of major fractions it is found that this test is satisfied by assigning an additional representative to a state when the population of the state divided by the harmonic mean of the state's present quota and its present quota plus one is greater than the similar figure for any other state.\(^{37}\) The

---

35. The apportionment of 1931 was made under the Permanent Apportionment Act, which authorized the use of either the method of major fractions or the method of equal proportions, and provided that if Congress did not make an apportionment, the method used in the last preceding apportionment should be put into effect. As chance would have it, equal proportions and major fractions gave identical results in 1931. Since major fractions had been used to make the last preceding apportionment, in 1911, it must technically be regarded as the method employed in 1931.

36. See p. 1380 infra.

37. Starting with the equation \(A/a = B/b\), the method of harmonic means proposes
The harmonic mean of two numbers is twice their product divided by their sum, so that the quantities by which state populations are divided to prepare rank indices are: \(2 \times \frac{1 \times 2}{1 + 2}, 2 \times \frac{2 \times 3}{2 + 3}, 2 \times \frac{3 \times 4}{3 + 4}, \ldots\).

The method of the harmonic mean was devised by Prof. Edward V. Huntington, of Harvard University, in 1921. It has never been used in an apportionment. As will be seen, it tends to favor slightly the smaller states, to reduce the absolute difference between the average district in the two states, \(A/a - B/b\), to the smallest possible quantity. If an additional seat is allotted to \(A\), this difference is \(B/b - A/(a + 1)\), while if \(B\) is given the added seat, the difference becomes \(A/a - B/(b + 1)\). The additional seat should be given to \(A\) if the absolute difference in that case is less than the absolute difference if the seat be given to \(B\), which may be expressed:

\[
\frac{B}{b} - \frac{A}{a + 1} < \frac{A}{a} - \frac{B}{b + 1}.
\]

Transposing this may be written:

\[
\frac{B}{b} + \frac{A}{a + 1} < \frac{A}{a} + \frac{B}{b + 1}.
\]

Reversing the inequality and factoring terms:

\[
A\left(\frac{1}{a} + \frac{1}{a + 1}\right) > B\left(\frac{1}{b} + \frac{1}{b + 1}\right).
\]

The harmonic mean \((H)\) of two numbers is twice their product divided by their sum. The harmonic mean of \(a\) and \(a + 1\) \((H \text{ of } a \text{ and } a + 1)\) is \(\frac{2(a)(a + 1)}{(a) + (a + 1)}\). The reciprocal is found to be:

\[
\frac{1}{H} = \frac{1}{\frac{2(a)(a + 1)}{(a) + (a + 1)}} = \frac{(a) + (a + 1)}{2(a)(a + 1)}.
\]

Multiplying each side of the equation by two:

\[
\frac{2}{H} = \frac{(a) + (a + 1)}{a(a + 1)} = \frac{a}{a(a + 1)} + \frac{a + 1}{a(a + 1)} = \frac{1}{a} + \frac{1}{a + 1}.
\]

The end result of this manipulation with the harmonic mean is seen to be identical with the factor being multiplied by \(A\) in the inequality developed above. Since the same result just demonstrated for the harmonic mean of \(a\) and \(a + 1\) can be demonstrated for the harmonic mean of \(b\) and \(b + 1\), the inequality may be written:

\[
\frac{2A}{H \text{ of } a \text{ and } a + 1} > \frac{2B}{H \text{ of } b \text{ and } b + 1}.
\]

Or finally, dividing by two:

\[
\frac{A}{H \text{ of } a \text{ and } a + 1} > \frac{B}{H \text{ of } b \text{ and } b + 1}.
\]

From this it is seen that if absolute differences in average districts are the criteria, a state should be awarded an additional representative whenever its population divided by the harmonic mean of the quota presently assigned it and the next higher quota is greater than the similar quantity for any other state.

38. See p. 1380 infra.
Method of Smallest Divisors

The method of smallest divisors takes the equation \( A/B = a/b \), with \( A \) being over-represented in comparison with \( B \), and transposes it to the form:
\[
a - b(A/B) = 0.
\]
The left-hand side of the equation is known as the absolute representation surplus, and the method of smallest divisors proposes so to apportion seats in the House that the absolute representation surplus between any two states cannot be reduced by transferring a representative from one state to the other. An eminent authority has termed this test “more artificial and less important” than the tests employed in the methods of major fractions, the harmonic mean, or equal proportions, but in many ways this test seems to be the method of examining the equality of an apportionment which would occur instinctively. The layman who knows that state \( A \) is twice as populous as state \( B \), and that state \( B \) has two representatives would be naturally inclined to multiply two by two, and look askance if state \( A \) were given five representatives instead of the four to which it seems entitled.

The test of the method of smaller divisors is satisfied by assigning an additional seat to a state when the population of the state divided by the quota already assigned it is greater than the population of any other state divided by the present quota of such other state. The divisors which are

39. See note 43 infra.
40. The representation surplus is in the form, \( a - b (A/B) \), or \( b - a (B/A) \). If an additional seat be allotted state \( A \), its surplus will be \( (a+1) - b (A/B) \), while if state \( B \) gets the seat, its surplus will be \( (b+1) - a (B/A) \). If the assignment of the seat to state \( A \) is correct, by the test of the method of smallest divisors, then:
\[
(a+1) - b (A/B) < (b+1) - a (B/A).
\]

Upon transposing terms the inequality becomes:
\[
(a+1) + a (B/A) < (b+1) + b (A/B).
\]
Reducing each side of the inequality to a common denominator:
\[
\frac{A + aA + aB}{A} < \frac{B + bA + bB}{B}.
\]
This may be simplified:
\[
1 + \frac{a (A+B)}{A} < 1 + \frac{b (A+B)}{B}.
\]
Subtracting unity from each side, and then dividing by \( (A+B) \):
\[
\frac{a}{A} < \frac{b}{B}.
\]

This expression may be turned upside down, with the inequality sign changing accordingly:
\[
\frac{A}{a} > \frac{B}{b}.
\]

Thus it is seen that an apportionment can be made which will minimize the absolute representation surplus if an additional seat be given a state whenever its population divided by its present quota exceeds the population of any other state divided by that state's present quota.
used to prepare the priority list for this method are, therefore, simply 1, 2, 3, .... The method of smallest divisors was devised by Professor Huntington in 1922, and has never been used in an apportionment. It tends to favor smaller states to an even greater extent than the method of the harmonic mean.41

**Method of Greatest Divisors**

The last of the four equations derived from the Fundamental Theorem, \( B/A = b/a \), is used by the method of greatest divisors, which proposes to minimize what it terms the absolute representation deficiency. The absolute representation deficiency is the quantity, \( a(B/A) - b \), when state B is under-represented relative to state A. When A is under-represented relative to B, the deficiency takes the form, \( b(A/B) - a \). Suppose, for example, that state A has a population of 1,200,000 and B a population of 600,000, while a proposed apportionment gives A five representatives and B only two. The absolute representation deficiency relative to state B is:

\[
5(600,000/1,200,000) - 2 = 5/2 - 2 = 1/2.
\]

If a representative were transferred from A's quota to B's, the deficiency relative to state A would be:

\[
3(1,200,000/600,000) - 4 = 6 - 4 = 2.
\]

Since the deficiency is less as the apportionment stands, the proposed transfer should not be made.

The test of the method of greatest divisors is satisfied by assigning an additional seat to a state when the population of the state divided by its present quota plus one is greater than the population of any other state divided by the present quota of such other state plus one.42 The divisors

41. See p. 1380 infra.
42. The representation deficiency is of the form, \( a(B/A) - b \), or \( b(A/B) - a \). If an additional seat be allotted state A the deficiency with regard to state B is \( (a + 1) (B/A) - b \), while if state B gets the seat, the deficiency with regard to state A will be \( (b + 1) (A/B) - a \). If the assignment of the seat to state A is correct, when measured by the test of minimal absolute representation deficiency, then:

\[
(a + 1) (B/A) - b < (b + 1) (A/B) - a.
\]

Subtracting unity from each side of the inequality:

\[
(a + 1) (B/A) - (b + 1) < (b + 1) (A/B) - (a + 1).
\]

Transposing terms:

\[
(a + 1) (B/A) + (a + 1) < (b + 1) (A/B) + (b + 1).
\]

Factoring each side of the inequality:

\[
(a + 1) \left( \frac{B}{A} + 1 \right) < (b + 1) \left( \frac{A}{B} + 1 \right).
\]

The second factor on each side may be placed over a common denominator:

\[
(a + 1) \left( \frac{A + B}{A} \right) < (b + 1) \left( \frac{A + B}{B} \right).
\]
used to prepare a priority list for this method of apportionment are, therefore, simply 2, 3, 4, . . .

The method of greatest divisors was devised by Prof. Victor d'Hondt, of the University of Ghent, in 1885. The test which it employs has been described, along with that of the method of smallest divisors, as "more artificial and less important" than that of the three other methods of apportionment; but, as was said in connection with the smallest divisors test, it seems to be a test which would occur instinctively to the average man. The greatest divisors method has never been used in an apportionment. It tends to favor larger states to an even greater extent than does the method of major fractions.44

**Method of Equal Proportions**

The four methods of apportionment which have been presented will each equalize representation by one of the four tests suggested by the Fundamental Theorem. A fifth method has been developed which gives an apportionment that satisfies any of the four tests, provided that relative differences are used to measure inequalities, rather than absolute differences. Starting from any of the four original equations, it is possible to demonstrate that relative differences as measured by any of the four tests of an apportionment can be minimized if an additional representative is given to a state when the population of the state divided by the geometric mean between its present quota and its present quota plus one is greater than that quantity for any other state.45 The geometric mean between two numbers is equal

\[ \frac{a+1}{A} < \frac{b+1}{B}. \]

And finally, taking the reciprocal of each side, and changing the inequality sign:

\[ \frac{A}{a+1} > \frac{B}{b+1}. \]

Thus it is seen that an apportionment can be made which will minimize the absolute representation deficiency if an additional seat be given a state whenever its population divided by its present quota plus one exceeds the population of any other state divided by the present quota plus one of such other state.

43. The comment that the tests of the methods of greatest divisors and smallest divisors are "more artificial and less important" than the tests of major fractions, harmonic mean, and equal proportions is made by Chafee, *supra* note 2, at 1028, n. 36. And Huntington, *op. cit. supra* note 2, at 14, 15, calls these tests "much more complicated and artificial than the tests for the three principal methods."

44. See p. 1380 infra.

45. The method of equal proportions may be developed by searching for that method of apportionment which will make the relative difference in the average districts minimal. The relative difference in the average districts of two states may be expressed as

\[ \frac{(B/b) - (A/a)}{(A/a)}. \]

For simplicity in development the following substitutions will be made:
to the square root of the product of the numbers, so that the divisors used to prepare a priority list for the method of equal proportions are:
\[ \sqrt{1 \times 2}, \sqrt{2 \times 3}, \sqrt{3 \times 4}, \ldots \]

The method of equal proportions was devised by Professor Huntington in 1920. It was used in the apportionment of 1941, and is the method prescribed by the Permanent Apportionment Act to be used unless Congress

the average district for state A \((A/a)\) will be termed \(x\); the average district for state B \((B/b)\) will be termed \(y\); the average district for state A if it is awarded an additional seat \((A/a + 1)\) will be termed \(x'\); the average district for state B if it is awarded an additional seat \((B/b + 1)\) will be termed \(y'\).

With these substitutions made, the relative difference in average districts between two states is \(\frac{y-x}{x}\). If state A is given an additional seat, the relative difference is \(\frac{y-x'}{x'}\), while if the additional seat goes to state B the relative difference is \(\frac{x-y'}{y'}\). In order to minimize the relative difference in average districts, an additional seat should be allotted to state A when:

\[ \frac{y-x'}{x'} < \frac{x-y'}{y'} \]

Upon separating the fractions this becomes:
\[ \frac{y}{x'} - 1 < \frac{x}{y'} - 1 \]

And since any number divided by itself equals unity:
\[ \frac{y}{x'} < \frac{x}{y'} \]

Adding unity to each side:
\[ \frac{y}{x'} < \frac{x}{y'} \]

Multiplying each side by \((x'y')\):
\[ \frac{y(x'y')}{x'} < \frac{x(x'y')}{y'} \]

Similar terms may be cancelled:
\[ y(y') < x(x') \]

The inequality can be reversed:
\[ x(x') > y(y') \]

By the substitution which was made:
\[ x = \frac{A}{a}; x' = \frac{A}{a+1}; y = \frac{B}{b}; y' = \frac{B}{b+1} \]

The inequality which has been developed may be translated back into the original terms:
makes an apportionment on its own. This method is thought to be impartial as between large and small states.\textsuperscript{45}

The application of the method of equal proportions can be shown by an example which arose in the apportionment following the 1940 census. The population of Arkansas was 1,949,398; the population of Michigan was 5,256,106. Controversy developed as to whether the 435th seat in the House should go to Arkansas, which already had been assigned six seats, or to Michigan, which had been assigned seventeen. The average districts and the individual shares under the two possible apportionments were as follows:

<table>
<thead>
<tr>
<th></th>
<th>Average Quota</th>
<th>Average District</th>
<th>Average Quota</th>
<th>Average District</th>
</tr>
</thead>
<tbody>
<tr>
<td>Arkansas</td>
<td>6</td>
<td>324,898</td>
<td>7</td>
<td>278,484</td>
</tr>
<tr>
<td>Michigan</td>
<td>18</td>
<td>292,006</td>
<td>17</td>
<td>309,183</td>
</tr>
</tbody>
</table>

| Absolute difference | 32,982 | 30,699 |
| Relative difference | 11.26% | 11.02% |

<table>
<thead>
<tr>
<th></th>
<th>Individual Quota</th>
<th>Individual Share</th>
</tr>
</thead>
<tbody>
<tr>
<td>Arkansas</td>
<td>6</td>
<td>3.0778</td>
</tr>
<tr>
<td>Michigan</td>
<td>18</td>
<td>3.4245</td>
</tr>
</tbody>
</table>

| Absolute difference | 0.3467 | 0.3559 |
| Relative difference | 11.26% | 11.02% |

The absolute difference between the average districts—the test used by the harmonic mean method—is least when the extra seat is given to Arkansas. The absolute difference between the individual shares—the test

\[
\frac{A}{a} \left( \frac{A}{a+1} \right) > \frac{B}{b} \left( \frac{B}{b+1} \right).
\]

Or:

\[
\frac{A^2}{a(a+1)} > \frac{B^2}{b(b+1)}.
\]

The square root of each side of the inequality can be taken:

\[
\frac{A}{\sqrt{a(a+1)}} > \frac{B}{\sqrt{b(b+1)}}.
\]

The geometric mean of two numbers is the square root of their product—precisely the type quantity in the denominators on each side of the inequality sign. Thus it is seen that if it is desired to minimize the relative difference between the average districts among the states, a seat should be given to a state when its population divided by the geometric mean of its present quota and its present quota plus one exceeds the similar quantity for any other state. And by a development similar to that here, it can readily be shown that the same method of apportionment will also minimize the relative difference between individual shares, the relative representation surplus, and the relative representation deficiency.

46. See p. 1380 \textit{infra}.
of the method of major fractions—is at a minimum when the extra seat is added to Michigan's quota. But no matter whether average districts or individual shares are compared, the relative difference is smallest when Arkansas gets a seventh seat, and this is the test used by the method of equal proportions. Similarly the relative differences would be smallest and the equal proportions test satisfied by giving the disputed seat to Arkansas if the absolute representation surpluses or deficiencies were compared.  

**Comparing The Modern Methods**

Each of the five modern methods of apportionment may be derived algebraically from the Fundamental Theorem of Congressional apportionment. Each of the methods involves a test which seems logically satisfying. Each of the methods permits an apportionment to be made by a simple priority list. None of the methods involves a paradox. But despite these strong arguments which can be made for any of the other four methods, the method of equal proportions seems clearly preferable. Its greatest advantage is that it gives an apportionment which is sound by any test, provided that relative differences are used, while no other method of apportionment will normally satisfy more than one test. It is difficult to defend an apportionment which seems fair when quotas are divided into populations but inequitable if populations are divided into quotas. All apportionments derived from methods other than equal proportions are subject to such contradictions.

And the method of equal proportions has the tremendous advantage of being neutral as between large states and small states, while each of the other four methods tends to favor either large or small states to varying extents. The results of apportionments by each of the five methods on the 1920 and 1930 census figures illustrate the extent of such tendencies:

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47. The seat was eventually given to Arkansas. See p. 1382 infra.
48. Although Chafee and Huntington would disagree with respect to the method of greatest divisors and the method of smallest divisors. See note 43 supra.
49. The method of equal proportions was recommended as preferable to any other method by the Advisory Committee to the Director of the Census in 1921, reprinted at 2 Hearings before Committee on the Census on H. R. 13471, 69th Cong., 2d Sess. 58 (1927), and by the National Academy of Sciences in 1929, reprinted at Hearings before the House Committee on the Census, 76th Cong., 3d Sess. 70–71 (1940).
51. The reason that reversing the quotient used as a measure leads to different results in the methods of major fractions and the harmonic mean, while leading to identical results when measured by equal proportions, is that the first two methods named involve, respectively, the arithmetic and harmonic mean, while the method of equal proportions is dependent on the geometric mean. The geometric mean is quite well behaved when turned upside down: the geometric mean of \( a \) and \( b \) is \( \sqrt{ab} \), and the geometric mean of \( \frac{1}{a} \) and \( \frac{1}{b} \) is \( \frac{1}{\sqrt{ab}} \). The arithmetic and harmonic means behave
very badly, however, when stood on their heads—an understandable shortcoming. The arithmetic mean of \( a \) and \( b \) is \( \frac{a+b}{2} \), but instead of the arithmetic mean of \( \frac{1}{a} \) and \( \frac{1}{b} \) being 

\[
\frac{2}{a+b},
\]

it is \( \frac{2ab}{a+b} \). The harmonic mean suffers from the same difficulty. This explains the versatility and ready reversibility of equal proportions.

52. The tables are taken from Schmeckebier, op. cit. supra note 2, at 65. A similar grouping of the results of each of the five methods in apportioning according to 1948 population estimates is as follows:

<table>
<thead>
<tr>
<th>State</th>
<th>Population</th>
<th>Smallest Divisors</th>
<th>Harmonic Mean</th>
<th>Equal Proportions</th>
<th>Major Fractions</th>
<th>Greatest Divisors</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>18,039,200</td>
<td>88</td>
<td>89</td>
<td>99</td>
<td>92</td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>491,000</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>490,600</td>
<td>3</td>
<td>3</td>
<td>2</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>D</td>
<td>489,600</td>
<td>3</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>E</td>
<td>489,400</td>
<td>3</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>20,000,000</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td></td>
</tr>
</tbody>
</table>

See Appendix B, infra, p. 1386.

Chafee, supra note 2, presents two interesting hypothetical examples which illustrate the varying tendencies of the modern methods. Id. at 1040, n. 63.
It has been argued that the discrimination of the method of major fractions in favor of large states is justifiable since it would help counteract the discrimination in favor of small states in the Senate. The clear wording of the Constitution refutes such an argument, and there is nothing in the debates of the Constitutional Convention, or in the first, or "constitutional" apportionment, to justify favoring large states in the House.

POLITICAL FACTORS IN APPORTIONMENT

Despite the mathematical superiority of the method of equal proportions, it would be naive to assume that its continued use by Congress is assured. It has previously been shown that in 1941 the method of equal proportions would have given Arkansas seven representatives and Michigan seventeen, while the method of major fractions would have allotted six to Arkansas and eighteen to Michigan. Congress chose at that time to give the extra seat to Arkansas, and thus to use the method of equal proportions. But this decision had nothing whatsoever to do with the mathematical or logical soundness of equal proportions. Arkansas is usually a safe Democratic state; Michigan's normal leanings are Republican. Every Democrat in Congress, except those from Michigan, voted for equal proportions and an extra seat for Arkansas. Every Republican voted for major fractions and an extra seat for Michigan. There were more Democrats in Congress than Republicans. Thus equal proportions.

The table in an appendix showing the results of each of the modern methods of apportionment when 1948 population estimates are used gives some insight into the political factors which may be operative in choosing a method of apportionment after the next census. Equal proportions and major fractions happen to give identical results, and the only difference between those two methods and the harmonic mean method is that the latter gives one more seat to Colorado, a politically undecided state, and one less seat to Texas, which is invariably Democratic. The method of smallest divisors

53. Schmeckeber, op. cit. supra note 2, at 72; Chafee, supra note 2, at 1041–2.
54. "A second-test . . . is suggested by the original object of apportionment. This was to give the more populous states the larger representation in the House to which their numbers were thought to entitle them and which they did not receive in the Senate. From the fundamental purpose of apportionment it follows that a method giving the populous states systematically either more members or fewer members per unit of population than the small states is incorrect. We have, then, two criteria, of a just and constitutional apportionment: . . . secondly, the method must hold the scales even between the large and the small states." Willcox, The Apportionment of Representatives, 6 Am. Econ. Rev. Supp. 3, 6–7 (1916).
55. See pp. 1379–80 supra.
57. See table, Appendix B, infra, p. 1386. The population figures on which the apportionment in the table is based are estimates by the Bureau of the Census of the population by states as of July 1, 1948, and are found in the New International Year Book 444 (1949).
also favors the Republicans, while the method of greatest divisors is even more favorable to the Democrats.\textsuperscript{53}

Another political factor which may be even more forceful than party loyalty is that of state pride. Very few representatives feel so versed in mathematics, or so devoted to their party, that they would be willing to return to their home state and try to explain a vote in favor of a method of apportionment which deprived that state of a seat or seats it otherwise would have gotten. This factor would seem to militate very strongly against acceptance by the House of an apportionment made by the method of smallest divisors or the harmonic mean, less strongly against the method of equal proportions and major fractions, and very strongly in favor of apportionment by the method of greatest divisors.\textsuperscript{59}

But even if the method of greatest divisors is used, state pride would still lend impetus to movements to increase the size of the House. Depending upon the method of apportionment employed, from eleven to seventeen states would lose seats in a House of 435. The amount of opposition to keeping the size of the House at that figure would be less if greatest divisors were used than if any other method were employed, but the opposition would still be strong enough to be significant.\textsuperscript{59}

On the assumption that where a state gains a seat, the extra seat will go to the party controlling the state legislature, and that where a state loses a seat, the seat will be taken, when possible, from the party in the minority in the legislature, and further assuming that the division between the parties would otherwise be identical with that in the present Congress, the political consequences of each of the methods of apportionment would be as follows:

<table>
<thead>
<tr>
<th>Method</th>
<th>Republicans</th>
<th>Democrats</th>
</tr>
</thead>
<tbody>
<tr>
<td>Present division</td>
<td>171</td>
<td>264</td>
</tr>
<tr>
<td>Smallest divisors</td>
<td>178</td>
<td>257</td>
</tr>
<tr>
<td>Harmonic mean</td>
<td>178</td>
<td>257</td>
</tr>
<tr>
<td>Equal proportions</td>
<td>177</td>
<td>258</td>
</tr>
<tr>
<td>Major fractions</td>
<td>177</td>
<td>258</td>
</tr>
<tr>
<td>Greatest divisors</td>
<td>175</td>
<td>260</td>
</tr>
</tbody>
</table>

Seventeen states have the same quota under any method of apportionment, and the state pride factor is therefore not present in their calculations. On the assumption that all of the present representatives from each of the other thirty-one states would vote for that method of apportionment which gives their state its largest quota, and against any method of apportionment which reduces their state's quota, the following numbers of votes are believed to be committed for and against each of the methods of apportionment:

<table>
<thead>
<tr>
<th>Method</th>
<th>For</th>
<th>Against</th>
</tr>
</thead>
<tbody>
<tr>
<td>Smallest divisors</td>
<td>40</td>
<td>185</td>
</tr>
<tr>
<td>Harmonic mean</td>
<td>16</td>
<td>163</td>
</tr>
<tr>
<td>Equal proportions</td>
<td>33</td>
<td>140</td>
</tr>
<tr>
<td>Major fractions</td>
<td>33</td>
<td>140</td>
</tr>
<tr>
<td>Greatest divisors</td>
<td>193</td>
<td>34</td>
</tr>
</tbody>
</table>

Eight states would lose a seat by any method of apportionment in a House of 435, and they would seem to provide a solid bloc of 76 votes committed to increasing the size of the House. The number of votes in the present House of the states which would lose a seat or seats by each of the methods of apportionment are as follows:

<table>
<thead>
<tr>
<th>Method</th>
<th>Votes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Smallest divisors</td>
<td>180</td>
</tr>
<tr>
<td>Harmonic mean</td>
<td>162</td>
</tr>
<tr>
<td>Equal proportions</td>
<td>165</td>
</tr>
<tr>
<td>Major fractions</td>
<td>166</td>
</tr>
<tr>
<td>Greatest divisors</td>
<td>104</td>
</tr>
</tbody>
</table>
CONCLUSION

The method of equal proportions is the best method of apportionment of the House yet devised. Most desirably, that method would be used to apportion the smallest House in which no state would lose a member. But the choice, after the 1950 census as after every census, is more likely to be dictated by considerations of political expediency and state pride. Indications are that at the next apportionment those factors would favor use of the method of greatest divisors, regardless of whether the size of the House is increased. When seats in Congress and electoral votes are at stake, the mathematician is subordinated to the politician.

APPENDIX A

CONSTRUCTION OF A PRIORITY LIST

The following method is used to construct a priority list by the method of greatest divisors for four states: A, with a population of 1,000,000; B, with a population of 750,000; C, with a population of 200,000; and D with a population of 100,000. The multipliers for the method of greatest divisors are found from a table to be:

<table>
<thead>
<tr>
<th>Multiplier</th>
<th>.5000 0000</th>
<th>.1111 1111</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>.3333 3333</td>
<td>.1000 0000</td>
</tr>
<tr>
<td></td>
<td>.2500 0000</td>
<td>.0909 0909</td>
</tr>
<tr>
<td></td>
<td>.2000 0000</td>
<td>.0833 3333</td>
</tr>
<tr>
<td></td>
<td>.1666 6667</td>
<td>.0769 2308</td>
</tr>
<tr>
<td></td>
<td>.1428 5714</td>
<td>.0714 2857</td>
</tr>
<tr>
<td></td>
<td>.1250 0000</td>
<td>.0666 6667</td>
</tr>
<tr>
<td></td>
<td>.1111 1111</td>
<td>.0625 0000</td>
</tr>
</tbody>
</table>

Multiplying the population of each state by each of these multipliers in turn gives lists which look like the following:

<table>
<thead>
<tr>
<th>State A</th>
<th>State B</th>
<th>State C</th>
<th>State D</th>
</tr>
</thead>
<tbody>
<tr>
<td>pop. 1,000,000</td>
<td>pop. 500,000</td>
<td>pop. 200,000</td>
<td>pop. 100,000</td>
</tr>
<tr>
<td>500,000</td>
<td>375,000</td>
<td>100,000</td>
<td>50,000</td>
</tr>
<tr>
<td>333,333</td>
<td>250,000</td>
<td>66,667</td>
<td>50,000</td>
</tr>
<tr>
<td>250,000</td>
<td>187,500</td>
<td>66,667</td>
<td>50,000</td>
</tr>
<tr>
<td>200,000</td>
<td>150,000</td>
<td>66,667</td>
<td>50,000</td>
</tr>
<tr>
<td>166,667</td>
<td>125,000</td>
<td>66,667</td>
<td>50,000</td>
</tr>
<tr>
<td>142,857</td>
<td>107,143</td>
<td>66,667</td>
<td>50,000</td>
</tr>
<tr>
<td>125,000</td>
<td>93,750</td>
<td>66,667</td>
<td>50,000</td>
</tr>
<tr>
<td>111,111</td>
<td>83,333</td>
<td>66,667</td>
<td>50,000</td>
</tr>
<tr>
<td>100,000</td>
<td>75,000</td>
<td>66,667</td>
<td>50,000</td>
</tr>
<tr>
<td>90,909</td>
<td>68,182</td>
<td>66,667</td>
<td>50,000</td>
</tr>
<tr>
<td>83,333</td>
<td>62,500</td>
<td>66,667</td>
<td>50,000</td>
</tr>
<tr>
<td>76,923</td>
<td>etc.</td>
<td>66,667</td>
<td>50,000</td>
</tr>
<tr>
<td>71,429</td>
<td>etc.</td>
<td>66,667</td>
<td>50,000</td>
</tr>
<tr>
<td>66,667</td>
<td>etc.</td>
<td>66,667</td>
<td>50,000</td>
</tr>
</tbody>
</table>

These rank indices are then arranged in one list in order of their size, after first allowing one representative for each state to comply with the constitutional requirement.
<table>
<thead>
<tr>
<th>Total number of seats so far assigned</th>
<th>Rank indices</th>
<th>State</th>
<th>Cumulative total of seats assigned to state</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>A</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>B</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>C</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>D</td>
<td>1</td>
</tr>
<tr>
<td>5</td>
<td>500,000</td>
<td>A</td>
<td>2</td>
</tr>
<tr>
<td>6</td>
<td>375,000</td>
<td>B</td>
<td>2</td>
</tr>
<tr>
<td>7</td>
<td>333,333</td>
<td>A</td>
<td>3</td>
</tr>
<tr>
<td>8</td>
<td>250,000</td>
<td>A</td>
<td>4</td>
</tr>
<tr>
<td>9</td>
<td>250,000</td>
<td>B</td>
<td>3</td>
</tr>
<tr>
<td>10</td>
<td>200,000</td>
<td>A</td>
<td>5</td>
</tr>
<tr>
<td>11</td>
<td>187,500</td>
<td>B</td>
<td>4</td>
</tr>
<tr>
<td>12</td>
<td>166,667</td>
<td>A</td>
<td>6</td>
</tr>
<tr>
<td>13</td>
<td>150,000</td>
<td>B</td>
<td>5</td>
</tr>
<tr>
<td>14</td>
<td>142,857</td>
<td>A</td>
<td>7</td>
</tr>
<tr>
<td>15</td>
<td>125,000</td>
<td>A</td>
<td>8</td>
</tr>
<tr>
<td>16</td>
<td>125,000</td>
<td>B</td>
<td>6</td>
</tr>
<tr>
<td>17</td>
<td>111,111</td>
<td>A</td>
<td>9</td>
</tr>
<tr>
<td>18</td>
<td>107,143</td>
<td>B</td>
<td>7</td>
</tr>
<tr>
<td>19</td>
<td>100,000</td>
<td>A</td>
<td>10</td>
</tr>
<tr>
<td>20</td>
<td>100,000</td>
<td>C</td>
<td>2</td>
</tr>
<tr>
<td>21</td>
<td>93,750</td>
<td>B</td>
<td>8</td>
</tr>
<tr>
<td>22</td>
<td>90,909</td>
<td>A</td>
<td>11</td>
</tr>
<tr>
<td>23</td>
<td>83,333</td>
<td>A</td>
<td>12</td>
</tr>
<tr>
<td>24</td>
<td>83,333</td>
<td>B</td>
<td>9</td>
</tr>
<tr>
<td>25</td>
<td>76,923</td>
<td>A</td>
<td>13</td>
</tr>
<tr>
<td>26</td>
<td>75,000</td>
<td>B</td>
<td>10</td>
</tr>
<tr>
<td>27</td>
<td>71,429</td>
<td>A</td>
<td>14</td>
</tr>
<tr>
<td>28</td>
<td>68,182</td>
<td>B</td>
<td>11</td>
</tr>
<tr>
<td>29</td>
<td>66,667</td>
<td>A</td>
<td>15</td>
</tr>
<tr>
<td>30</td>
<td>66,667</td>
<td>C</td>
<td>3</td>
</tr>
</tbody>
</table>

If the population of this hypothetical nation is 2,050,000, the natural ratio for a House of 20 seats is 102,500. When a priority list is used, the artificial ratio may be any figure between the rank index opposite the number of the total House and the rank index for the next largest size of the House. In this case any number between 100,000 and 93,750 will suffice as an artificial ratio, but the customary procedure is to choose the figure halfway between the two rank indices, which in this case is 96,875. If this artificial ratio is divided into the population of any state, the number of seats which the state should be given in a House of 20 will be the whole number in the exact quota, the fractions being disregarded. Analogous principles apply to the artificial ratios derived from the other modern apportionment methods.
## Appendix B

### Estimated Quota Computed from 1948 Population by:

#### State

<table>
<thead>
<tr>
<th>State</th>
<th>Estimated Population 1948</th>
<th>Present Allotment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alabama</td>
<td>2,848,000</td>
<td>9</td>
</tr>
<tr>
<td>Arizona</td>
<td>664,000</td>
<td>2</td>
</tr>
<tr>
<td>Arkansas</td>
<td>1,923,000</td>
<td>7</td>
</tr>
<tr>
<td>California</td>
<td>10,031,000</td>
<td>23</td>
</tr>
<tr>
<td>Colorado</td>
<td>1,165,000</td>
<td>4</td>
</tr>
<tr>
<td>Connecticut</td>
<td>2,011,000</td>
<td>6</td>
</tr>
<tr>
<td>Delaware</td>
<td>297,000</td>
<td>1</td>
</tr>
<tr>
<td>Florida</td>
<td>2,356,000</td>
<td>6</td>
</tr>
<tr>
<td>Georgia</td>
<td>3,128,000</td>
<td>10</td>
</tr>
<tr>
<td>Idaho</td>
<td>530,000</td>
<td>2</td>
</tr>
<tr>
<td>Illinois</td>
<td>8,670,000</td>
<td>26</td>
</tr>
<tr>
<td>Indiana</td>
<td>3,909,000</td>
<td>11</td>
</tr>
<tr>
<td>Iowa</td>
<td>2,625,000</td>
<td>8</td>
</tr>
<tr>
<td>Kansas</td>
<td>1,968,000</td>
<td>6</td>
</tr>
<tr>
<td>Kentucky</td>
<td>2,819,000</td>
<td>9</td>
</tr>
<tr>
<td>Louisiana</td>
<td>2,576,000</td>
<td>8</td>
</tr>
<tr>
<td>Maine</td>
<td>900,000</td>
<td>3</td>
</tr>
<tr>
<td>Maryland</td>
<td>2,148,000</td>
<td>6</td>
</tr>
<tr>
<td>Massachusetts</td>
<td>4,718,000</td>
<td>14</td>
</tr>
<tr>
<td>Michigan</td>
<td>6,195,000</td>
<td>17</td>
</tr>
<tr>
<td>Minnesota</td>
<td>2,940,000</td>
<td>9</td>
</tr>
<tr>
<td>Mississippi</td>
<td>2,121,000</td>
<td>7</td>
</tr>
<tr>
<td>Missouri</td>
<td>3,947,000</td>
<td>13</td>
</tr>
<tr>
<td>Montana</td>
<td>511,000</td>
<td>2</td>
</tr>
<tr>
<td>Nebraska</td>
<td>1,301,000</td>
<td>4</td>
</tr>
<tr>
<td>Nevada</td>
<td>142,000</td>
<td>1</td>
</tr>
<tr>
<td>New Hampshire</td>
<td>548,000</td>
<td>2</td>
</tr>
<tr>
<td>New Jersey</td>
<td>4,729,000</td>
<td>14</td>
</tr>
<tr>
<td>New Mexico</td>
<td>571,000</td>
<td>2</td>
</tr>
<tr>
<td>New York</td>
<td>14,386,000</td>
<td>45</td>
</tr>
<tr>
<td>North Carolina</td>
<td>3,715,000</td>
<td>12</td>
</tr>
<tr>
<td>North Dakota</td>
<td>3,560,000</td>
<td>2</td>
</tr>
<tr>
<td>Ohio</td>
<td>7,799,000</td>
<td>23</td>
</tr>
<tr>
<td>Oklahoma</td>
<td>2,362,000</td>
<td>8</td>
</tr>
<tr>
<td>Oregon</td>
<td>1,626,000</td>
<td>4</td>
</tr>
<tr>
<td>Pennsylvania</td>
<td>10,689,000</td>
<td>33</td>
</tr>
<tr>
<td>Rhode Island</td>
<td>748,000</td>
<td>2</td>
</tr>
<tr>
<td>South Carolina</td>
<td>1,991,000</td>
<td>6</td>
</tr>
<tr>
<td>South Dakota</td>
<td>623,000</td>
<td>2</td>
</tr>
<tr>
<td>Tennessee</td>
<td>3,149,000</td>
<td>10</td>
</tr>
<tr>
<td>Texas</td>
<td>7,230,000</td>
<td>21</td>
</tr>
<tr>
<td>Utah</td>
<td>635,000</td>
<td>2</td>
</tr>
<tr>
<td>Vermont</td>
<td>374,000</td>
<td>1</td>
</tr>
<tr>
<td>Virginia</td>
<td>3,029,000</td>
<td>9</td>
</tr>
<tr>
<td>Washington</td>
<td>2,487,000</td>
<td>6</td>
</tr>
<tr>
<td>West Virginia</td>
<td>1,915,000</td>
<td>6</td>
</tr>
<tr>
<td>Wisconsin</td>
<td>3,309,000</td>
<td>10</td>
</tr>
<tr>
<td>Wyoming</td>
<td>275,000</td>
<td>1</td>
</tr>
</tbody>
</table>

### Quota Computed from 1948 Population by:

- **Smallest Divisors**
- **Harmonic Mean**
- **Equal Proportions**
- **Major Divisors**
- **Greatest Divisors**

#### Number of states with:

- **Increased quota**
- **Decreased quota**
- **Unchanged quota**

<table>
<thead>
<tr>
<th>Increased quota</th>
<th>Decreased quota</th>
<th>Unchanged quota</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td>11</td>
<td>28</td>
</tr>
<tr>
<td>6</td>
<td>11</td>
<td>31</td>
</tr>
<tr>
<td>8</td>
<td>12</td>
<td>29</td>
</tr>
<tr>
<td>8</td>
<td>12</td>
<td>29</td>
</tr>
<tr>
<td>8</td>
<td>12</td>
<td>29</td>
</tr>
</tbody>
</table>