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This year, the next decennial United States census has begun. When the results of the new census are released, nearly every state may be forced to redraw the boundary lines defining its state and federal legislative districts. The goals and methods a state adopts for political redistricting pose important policy questions for the state. The location of the boundary lines will greatly influence the composition of the legislatures, and the new legislatures will, in turn, affect the direction of states for years into the future. Since the U.S. Supreme Court first held political redistricting to be justiciable in *Baker v. Carr*,[1] redistricting has been constrained by legal decisions defining constitutional requirements for drawing district lines.[2] Political redistricting has thus become a question of law as well as policy.

Recently, redistricting has also become a new playground for computers. The improvements in computer technology made during the past decade will allow political groups to use computers in order to construct carefully gerrymandered districting plans; redistricters will be able to draw plans "manually" and use the computer to measure quickly the political impact such plans would have.[3] As implied by the term gerrymander, this new use of computers as "bookkeepers" in redistricting can be expected to produce district boundaries that directly benefit those drawing the lines and may strongly decrease the value of many state residents' votes. One can expect that these new plans, though gerrymandered, will meet the

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1. 369 U.S. 186 (1962).

2. For example, Karcher v. Daggett, 462 U.S. 725 (1983), struck down as unconstitutional a plan for New Jersey's U.S. congressional districts because the average deviation from perfect population equality across districts was 0.1384%. Since *Baker*, redistricting plans like the plan in *Karcher* may be found to violate the equal protection clause, U.S. CONST. amend. XIV, § 1.

3. *New Age of Gerrymandering: Political Magic by Computer?*, N.Y. Times, Jan. 10, 1989, at A1, col. 5 [hereinafter *Age of Gerrymandering*]; *see also* Note, *Computer Models and Post-Bandemer Redistricting*, 99 YALE L.J. (forthcoming 1990) (discussing current use of computers in political redistricting).

population equality standards used by courts in the past to reject redistricting plans.

This new use of computers in redistricting, along with the 1986 *Davis v. Bandemer*[4] decision holding partisan gerrymandering justiciable for the first time, should prompt new changes in political redistricting law and policy. In fact, some expect that after the 1990 census, virtually every proposed redistricting plan might be challenged in the courts.[5]

In response to a perceived need for changes in political redistricting for the future, this Current Topic proposes a new method for redistricting. Specifically, this Current Topic proposes a new computer model that might be used to automate redistricting. This Current Topic does not address the merits of automated redistricting as such, except to the degree that readers find the logic of automated redistricting inherently compelling. Instead, it proposes that a particular, new type of computer algorithm—simulated annealing—be examined for use in political redistricting.[6]

The Current Topic will start with the proposition, first advanced in the 1960's, that automated redistricting models should be used to create district boundary lines.[7] Unlike the *manual* use of computers in redistricting, where politicians or their agents draw the political boundary lines by hand, checking the political or other makeup of the resulting map using computer databases,[8] *automated* redistricting forces line drawers to specify their goals for redistricting in advance, while the computer program does the actual line drawing. The goals that might be specified include criteria such as maximizing the compactness of districts, making districts equal in population, and

4. 478 U.S. 109 (1986) (plurality opinion). In *Bandemer*, the Supreme Court, in examining redistricting plans for the state of Indiana, found that it would be possible for a gerrymander to discriminate unconstitutionally against members of a political party—a new equal protection argument. The Court did not find that the Indiana plan constituted an unlawful gerrymander, but the decision cleared the way for a new class of plaintiffs to challenge redistricting plans.

5. *Age of Gerrymandering*, *supra* note 3.

6. For an extensive discussion of *why* computer models should be used in redistricting, see Note, *supra* note 3.

7. *See, e.g.*, Nagel, *Simplified Bipartisan Computer Redistricting*, 17 STAN L. REV. 863 (1965); Weaver & Hess, *A Procedure for Nonpartisan Districting: Development of Computer Techniques*, 73 YALE L.J. 288 (1963); *Symposium on Computers & Reapportionment*, 2 RUTGERS J. COMPUTERS TECH. & L. 13 (1971).

8. This is the threatened use of computers indicated in *Age of Gerrymandering*, *supra* note 3. The use of computers in this situation can clearly aggravate the tendency to gerrymander.

maintaining the contiguity of districts.[9] The modelers would also specify the types of input they were using in the model, such as census data and collected political data like the political makeup of city blocks. Given such input data and a redistricting algorithm, a computer model would then produce a plan for political boundary lines.

Although the notion of automated redistricting was first discussed more than twenty-five years ago, recent advances in computer power and algorithm design suggest that radical improvements over past redistricting algorithms may now be possible. Because automated redistricting could be an important policy solution to many redistricting problems, the public must be made aware of the possible positive uses of computers in redistricting given today's technology, before forming irreversible, negative views of computerized redistricting that may follow from seeing the "computer as bookkeeper" method used to create unsatisfactory, gerrymandered political districts. Additionally, if people continue to view automated redistricting only in terms of dated computer algorithms, they may never consider automated redistricting to be a viable policy solution to a difficult problem.

I. Automating Redistricting

A. Terminology

It may be helpful to explain some terminology that must be used in describing automated redistricting.

1. Algorithms. An algorithm is a step-by-step process defining *exactly* how a task is to be accomplished. For example, an algorithm for opening a door may be: Go to door. Move hand to doorknob. Grasp doorknob. Turn doorknob clockwise. If this is a "push" door, then push door open. If this is a "pull" door, then pull door open. A redistricting algorithm would define the steps telling exactly how to rearrange the information provided to a computer (the data) into a redistricting plan.

2. Computer Programs. A redistricting algorithm must in turn be translated into a computer program, consisting of lines of computer code, that would tell the computer to perform the tasks described by the redistricting algorithm. A computer program is a

9. For lists of criteria for political redistricting and definitions, see, e.g., Lijphart, *Comparative Perspectives on Fair Representation: The Plurality-Majority Rule, Geographical Districting, and Alternative Electoral Arrangements*, in REPRESENTATION AND REDISTRICTING ISSUES 145-46 (1982).

translation of a theoretical idea (an algorithm) into a form usable by a computer.

3. Computer Models. Finally, think of the overall computer model as a book: The plot might be comparable to an algorithm, as in "boy meets girl." The written words on the page are comparable to a computer program, since they translate the plot (algorithm) into a form that we (the computer) can read. And the term "computer model" reflects the whole book, bringing to mind not only the story (algorithm), but also the words (program), and the images those words create for our imagination (the notion of political redistricting itself, along with possible redistricting plans and concerns).

B. Representing the Redistricting Problem Mathematically

Before proposing a new algorithm for political redistricting, it is important to clarify mathematically how the process of redistricting is reduced to a computer model. In general, a computerized redistricting scheme takes an initial redistricting plan as input and improves it using an algorithm, producing a new plan. Input data for the model will include a list of the smallest "enumeration districts" (EDs) in a state that can be moved; for example, in a city, an enumeration district might be a city block. A redistricting program thus could rearrange collections of city blocks into districts, but a city block could not be broken into smaller pieces. In sparsely populated regions, towns or counties might be the enumeration districts. Input data would also include a "touchlist," supplied to tell the program which EDs are neighbors in order to have the model test for contiguity.[10]

The automated schemes considered here allow the modeler to choose which criteria, such as population criteria or compactness (a measure distinguishing snaky districts from, say, round ones), should be improved.[11] Most of the potentially desirable criteria for redistricting can be adapted in a computer model via either the objective function (such as, "maximize compactness"),[12] the constraints (such as keeping populations equal in districts, and ensuring that each district is contiguous), the data structure (for example, treating two neighboring EDs separated by a mountain range as

10. EDs and touchlists are discussed extensively in Weaver & Hess, *supra* note 7.
11. *See* Lijphart, *supra* note 9, for a discussion of possible criteria for redistricting.
12. For example, goals or objective functions might be "Maximize compactness!" for a redistricting algorithm, or "Start car!" for someone going through the steps required to start a car.

noncontiguous),[13] or the program structure (establishing what goals the program will look at and in what order).[14]

As an example of the great flexibility modelers have in designing an automated redistricting model, consider a simple case. Recall that the touchlist is an array of numbers that tells the computer program which EDs neighbor other EDs. So, for example, if two towns are neighbors, their touchlist value should be 1, while if they do not touch, their value should be 0. Just as a touchlist value could be set at 0 if two towns neighbor but are separated by a river, a touchlist value could be placed at, say, 0.5 if towns touch but their socioeconomic conditions differ. Inclusion of factors such as these that increase the modeler's control of the results is *not* meant to suggest that added control is desirable, but only to show the flexibility that can be built into a computer model. Adjusting touchlists in this manner might represent a compromise to make legislators more amenable to the use of computers. Legislators would still be accountable for their choices because the adjusted touchlists would be available to the courts and the public. For example, courts could regulate the redistricting process by finding that certain touchlist choices represent an unconstitutional intent to gerrymander, or unconstitutional discrimination.

C. *Choosing a Plan*

Unlike the "computer as bookkeeper" method, the automated process suggested here does not allow for human interaction in the actual redistricting procedure; once an algorithm and the input data are chosen, a model will produce a single districting plan. Legislators, and possibly judges, must consider whether to produce a single plan for immediate implementation, or whether, instead, to run the model several times using different input data in order to produce more than one plan from which to choose. If, for example, the results of an algorithm depend on the initial data, feeding the computer several different initial schemes should produce a set of plans

13. The data structure can essentially be used to tell the computer what information it can manipulate. This use of the term "data structure" should not necessarily imply the same connotations that "data structure" ordinarily has in computer science usage.

14. For example, if the goal of redistricting were to create constitutionally sound districts that were changed as little as possible from previous districts, the program itself might be designed in such a way as to provide a least changed plan. *See* Nagel, *supra* note 7.

from which to choose.[15] Or, if a program sequentially examines the list of EDs when improving districts, running the model after changing the order of the input should produce different results.[16]

Individuals might prefer to use the computer to produce a set of alternative plans if they were hesitant to entrust redistricting completely to a machine. However, to do so arguably would defeat the purpose of using a computer to redistrict. Allowing politicians to choose among plans without requiring that they justify their decisions essentially would remove the accountability factor which is the very strength of computerized redistricting. Politicians could try to use a computer to "legitimate" gerrymandering; the party in power could produce plans until it found a desirable scheme; or, if allowed only a limited number of plans, the majority party could simply choose the plan most beneficial to it. Likewise, the incumbents could choose the plan most beneficial to them.

Because it is ultimately desirable to have an automated model produce a *single* redistricting plan, it is important that the final plan be defensible. If the computer model simply chose randomly among equal population plans, its opponents could argue that the automated process was really no more rational than the present system, and that the only improvement of automation was the removal of intent to gerrymander. In fact, there are at least two standards which could be used to defend the computer's selection of a plan—having the model produce the *least changed plan* which meets population equality,[17] or claiming that the algorithm's solution is optimal.

Optimality of the solution would seem to be the strongest possible defense of a computer-generated redistricting plan. However, it is not clear at first that one can claim that any redistricting solution is optimal. For example, in any state, many plans exist which satisfy equality of population in districts, so no plan could be called "optimal" just by virtue of the fact that it met the criterion of population equality. However, "the more criteria [there are in redistricting], the fewer the solutions that can satisfy them."[18] Additional research

15. *See, e.g.*, Weaver & Hess, *supra* note 7, at 302-04. The Weaver/Hess algorithm will produce different results for different initial guesses of the location of the population centers for districts.

16. *See, e.g.*, Nagel, *supra* note 7, at 885 ("A simple way of trying to reach the true [optimal solution] is to run the same set of data cards a few times with the cards arranged in a different random order each time.").

17. This goal is one of the considerations in the Nagel model. *See* Nagel, *supra* note 7, at 894.

18. Papayanopoulos, *Quantitative Principles Underlying Apportionment Methods*, 219 ANNALS N.Y. ACAD. SCI. 181, 187 (1973).

might suggest, for example, the existence of a unique optimal solution for a compact, contiguous, equal population plan in a state. While the existence of an optimal solution is a function of the criteria the modelers choose to include in the redistricting program, this avenue seems worthy of additional investigation.[19]

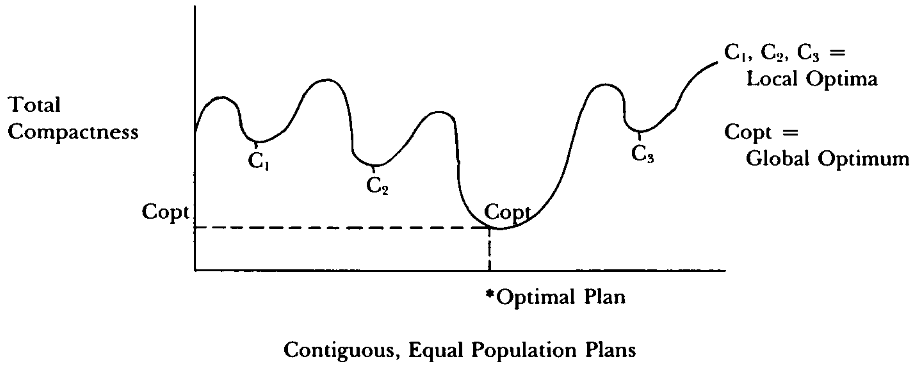
D. *The Notion of Optimality*

The notion of optimality in redistricting might be easier to understand if illustrated by a less complex example. Consider a street full of potholes. If a child wants her toy to fall as deep into the earth as possible, she might throw the toy into this street. The toy might roll or bounce, and end up in one of the potholes. The final resting place of the toy might be one of the more shallow potholes, in which case one would call the solution a "local optimum," which is optimal in the sense that the toy is as deep as it can be anywhere in its immediate neighborhood. Not only is the toy lower than it would be if it had stayed on the street, but at the bottom of the shallow pothole it is also deeper than it could be if it got stuck halfway down that pothole. If the child actually succeeded in placing the toy into the deepest pothole in the street, one would call the result the "global optimum." In the context of redistricting, a local optimum would be good, but a global optimum would be defensible as the optimal solution to a problem. The sketch below illustrates the concept of local and global optima in political redistricting:

19. Notice that if Nagel's method needs to be adjusted to consider compactness at the same time as population equality, then the standard of overall optimality of the plan might be a more appropriate test for Nagel's model than the least changed plan would be, since it would otherwise not be clear when to stop adjusting compactness. *Cf.* Nagel, *supra* note 7, at 885.

Notice, too, that Nagel's use of the least changed plan is one example of creating the possibility of an optimal solution by establishing the criteria in advance. When the criterion is "changing plan the least," the optimal solution will be the least changed plan.

An Example of Local versus Global Optima in Political Redistricting



Notice that this example has been simplified. First, it assumes that it is possible to express the total compactness of a plan with a single measure by, for example, considering only the sum of compactness in the plan. The same liberty has been taken with population equality. This sketch is also simplified in that it represents compactness of districts as a continuous and continuously differentiable function. Realistically, the line shown should only be a series of discrete points.

In the pothole example, the algorithm is the procedure making the toy bounce or roll along the street. In redistricting, the toy is replaced by an objective function, such as the measure of the district's population equality. Measuring population equality alone, the procedure might end at any one of many equally shallow "potholes." However, as one adds other criteria, such as compactness, these apparently equal potholes have different depths. For example, the objective function might measure compactness as well as population equality, in which case there may well be a unique optimal solution to the problem.

An important consideration in the use of an automated process to produce a single redistricting plan is whether an algorithm can actually locate a defensible, globally optimal plan.[20] One could argue that an automated process that cannot find a true global optimum should only be used to produce several plans from which legislators could choose a districting scheme, because the modeler would not

20. The Weaver/Hess algorithm, for example, suggested a method to find compact, equal population districts. However, not only was the method unable to test for contiguity, but also the integer programming solution used was dependent on the initial input chosen; a computer solution dependent on initial conditions is a characteristic of an algorithm that can only test for local optima. *See* Weaver & Hess, *supra* note 7, at 302-04.

be able to justify the process's otherwise random selection of one suboptimal plan.[21] If an algorithm could be designed to find a single global optimal solution,[22] a completely automated redistricting procedure might be both justifiable and desirable.

Unfortunately, computerized redistricting represents a very difficult optimization problem because it is a combinatorics problem. Combinatorics problems involve rearranging pieces into different combinations—for example, rearranging EDs into political districts.[23] As one author discussing the combinatorial nature of the redistricting problem commented:

> some of the most elusive problems of mathematical programming are combinatorial. Until a precise, highly efficient solution method is developed for a problem of this type, many lesser techniques may be explored. Some may work under certain circumstances and not under others. Some may be faster but less accurate than others, and so on.[24]

With one exception, the techniques developed thus far for political redistricting have been unable to find an optimal solution.[25] The exception, complete enumeration of all possible arrangements of EDs into districts,[26] is impossible to use in cases having more

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21. However, it might also be reasonable to have the computer automatically choose a single suboptimal plan if that plan represents the best effort that technology can presently achieve, subject to time and money constraints. Such a selection could be viewed using Herbert Simon's notion of "satisficing," which recognizes that while it would often require too much information to "optimize" real-life situations mathematically, it is still possible to make good decisions *subject to* the environmental and informational constraints of the decision. *See, e.g.*, H. SIMON, ADMINISTRATIVE BEHAVIOR: A STUDY OF DECISION-MAKING PROCESSES IN ADMINISTRATIVE ORGANIZATION 240-44, 272 (3d ed. 1976).

22. Given the previous discussion of satisficing, *supra* note 21, the global optimal for the purposes of this Current Topic should really be taken to mean the "best optimal" that can be found using today's technology.

23. Most solvable optimization problems are not combinatoric problems, but instead are polynomial problems, which are formulations that take a computer on the order of, say n, or n-squared, or n-raised to the kth power, steps to solve. A combinatorics problem cannot be described in one of these polynomial forms, and instead usually takes on the order of some number-raised to the nth power, steps to solve.

In an example of a polynomial problem, if a problem takes n-raised to the second operations to solve, then for n=10, the problem might take .001 seconds to solve, and sextupling n to 60 would raise the time of solution to .036 seconds (.001 * 6-squared). In contrast, for the simplest nonpolynomial algorithm, taking 2-raised to the n steps to solve, if n=10 takes the computer .001 seconds to solve, n=60 would take 366 centuries. *See* M. GAREY & D. JOHNSON, COMPUTERS AND INTRACTABILITY: A GUIDE TO THE THEORY OF NP-COMPLETENESS 6-9 (1979). Garey and Johnson describe a problem as "*intractable* if it is so hard that no polynomial time algorithm can possibly solve it." *Id.* at 8.

24. Papayanopoulos, *supra* note 18, at 188.

25. *See id.* at 188-89.

26. *See, e.g.*, PARR, THE EQUALIZER (Apple Version 1.0; IBM Version 1.0. Distributed by National Collegiate Software Clearinghouse, School of Humanities and Social Sciences, North Carolina State University) (on file with author).

than approximately 50 EDs because of the explosive nature of combinatorics problems; while enumeration might be helpful in redrawing boundaries for town council seats, this method is not promising for the redistricting of major states.

II. The Proposal: Simulated Annealing, A State-of-the-Art Technique

While former computer models have not been able to find defensible, optimal redistricting solutions, a recently practicable computer technique called simulated annealing may be well suited for solving the political redistricting problem. In science, annealing describes the way in which hot metal is cooled slowly, allowing molecules to arrange themselves into a stable, low-energy configuration. Metal cooled in this way is stronger than metal which is cooled quickly, trapping molecules in high-energy, less stable configurations. *Simulated* annealing offers computer scientists an opportunity to use this principle of slow cooling to find the optimal solutions to complex problems. In the pothole example, annealing would enable an automated toy to move along the street in a pattern more likely to help it find the deepest pothole. Simulated annealing simply describes a different algorithm for solving mathematical problems. As a 1986 text commented, "[t]he method of simulated annealing is a technique that has recently attracted significant attention as suitable for optimization problems of very large scale."[27]

The details of annealing are somewhat beyond the scope of this discussion. In summary, however, if one starts with any random configuration of a system (for example, think of the current redistricting plan for a state as the initial configuration) and then tries to change random pieces of the system (for example, moving groups of EDs between legislative districts), always keeping changes that improve the system (for example, making districts closer in equality of population), and occasionally keeping changes which harm the system (for example, making districts less equal in population), then eventually one should reach an optimal or nearly optimal configuration of the system (a plan that meets the objective function, such as the most compact plan).[28]

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27. W. PRESS, B. FLANNERY, S. TEUKOLSKY & W. VETTERLING, NUMERICAL RECIPES: THE ART OF SCIENTIFIC COMPUTING 326 (1986) [hereinafter PRESS] (emphasis omitted).

28. For an excellent description of annealing with examples, see PRESS, *supra* note 27, at 326-34. In order to understand the power of the annealing technique, it is important to note that the problem on which Press demonstrates annealing is an example of a difficult nonpolynomial problem. *See* discussion and definition of nonpolynomial problems, *supra* note 23.

Simulated Annealing

This technique's emphasis on randomization should produce a plan that is independent of the initial conditions chosen. As a classic text on numerical methods explains, annealing is designed to overcome the shortcomings of most optimization techniques, which "[go] greedily for the quick, nearby solution . . . [leading] to a local, but not necessarily a global, minimum."[29]

To apply simulated annealing and optimization to the redistricting problem, one must first consider the initial configuration and "energy" aspects of the model. Initially, one would assign the EDs in the state to districts according to the current map. The "energy" of this system might be defined as:

Energy = Compactness + Contiguity + Population Equality[30]

The goal is to minimize the energy of the system, just as the molecules in annealing metal try to reach their minimum energy configuration. Energy in this sense is essentially another name for an objective function. Notice that the formulation of this objective function using selected criteria was chosen *only* as an example. Again, most criteria of interest in redistricting can be included in some aspect of the model.[31] Notice, too, that one can minimize energy in order to either minimize or maximize terms in the model, because minimizing any value is the same as maximizing the exact opposite of that value. Maximization and minimization are to this degree interchangeable.

One can view the model above as an optimization of compactness, with the other terms included in the equation as a "relaxation" of a

29. PRESS, *supra* note 27, at 327. Most optimizing algorithms, trying to reach the "quick, nearby solution," are analogous to the example in real annealing in which metal is cooled quickly, leaving molecules in a suboptimal configuration.

30. Expressed mathematically,

$$\text{Energy} = \sum_{\substack{\text{districts}\\ i=1}}^{k} \sum_{\substack{\text{EDS}\\ j\epsilon i}} \text{population}_j\,(\text{distance from center of } j \text{ to population-center of } i)^2 +$$

$$\lambda_1 \sum_{\substack{\text{districts}\\ i=1}}^{k} \begin{bmatrix} 0 \text{ if district is contiguous} \\ 1 \text{ otherwise} \end{bmatrix} + \lambda_2 \sum_{\substack{\text{districts}\\ i=1}}^{k} \left| \left(\sum_{\substack{\text{EDS}\\ j\epsilon i}} \text{population}_j \right) - \frac{1}{K}\,(\text{total state population}) \right|$$

Notice that this is only one formulation of the redistricting problem into a mathematical model. Computer modeling is an extremely flexible form. *See supra* Section I.B. A model could include additional terms that are different from these, such as a term designed to protect incumbents; a model could also use different representations of included terms, such as a different measure of compactness, say, to maximize the compactness of the least compact district, as opposed to a measure like the current one designed to maximize the average compactness.

31. *See supra* Section I.B.

constrained optimization problem. Relaxation provides a means of solving a constrained problem as an unconstrained one, by allowing one to bring the constraints of the problem into the objective function. In this example, it is the requirements of equal population and contiguity which provide constraints on the otherwise unconstrained problem of maximizing compactness of districts. A simple, real-world problem, while not completely analogous, may explain the notion of relaxation. Consider a student in a cafeteria line with a simple objective function: Maximize food intake. He may think of this objective as a need to pile as much food onto his tray as possible. However, the student may be under a constraint: He must leave room for a tasty dessert. The student now has a constrained optimization problem. He must pile as much food on his tray as possible, but he knows in the back of his mind that he must leave room for dessert. Our student may simplify this problem by relaxing it: The student can draw a line on his tray, so that one quarter of his tray is left clear for desserts, and the remaining portion of his tray is free to be piled with food. The student has then created an *unconstrained* optimization problem. He will pile as much food onto the remaining three-quarters of the tray as he wants, without worrying about his constraint, since he has "absorbed" the constraint into the objective function. This can be thought of as a relaxation.

Because redistricting represents a discrete,[32] constrained,[33] nonlinear,[34] multiattribute problem,[35] and because simulated annealing works well on discrete, nonlinear, multiattribute problems, simulated annealing should be able to handle redistricting using the reformulation provided here. By absorbing constraints such as population equality and contiguity into the objective function, the relaxation allows one to solve an unconstrained optimization that is, as a rule, much easier to solve than a constrained optimization problem. The constraints are absorbed by bringing them into the energy equation with weights or multipliers[36] that determine how much the

32. Discrete, that is, as opposed to continuous. EDs are building blocks that are whole entities and must be treated as discrete entities.

33. Constrained because the Court requires districts to be of equal population.

34. Nonlinear because, for example, two EDs do not necessarily have twice the population and twice the size of a single ED.

35. Multiattribute because the problem has multiple goals. *See* Lijphart, *supra* note 9.

36. Here the multipliers are the lambda terms in the previous equation, *supra* note 30. These multipliers help the relaxation. For example, in the cafeteria example, the student set aside one quarter of his cafeteria tray for dessert. But he could just have easily set aside a half or an eighth of the tray for dessert. Although an extensive discussion of Lagrangian multipliers is well beyond the scope of this Current Topic, in the

energy term is penalized when the constraint is not met; the multipliers should be chosen to force the program to meet the constraints.[37]

The main portion of the annealing process will choose EDs at random and change their districting assignments, either swapping EDs or moving single EDs.[38] If the new assignment leads to a lowering of the energy of the system, the districting assignments will be updated. If the new assignment raises the energy of the system, the change may or may not be implemented. After the process runs for a long time, the system should reach a relatively stable condition. When annealing is finally stopped, the districting assignments should represent a defensible optimal or nearly optimal configuration.

III. Policy Choices and the Formulation of the Model

Notice that the construction of the model leaves to the modeler many choices. Designers choose not only the criteria to be considered in the model, but also the multipliers or weights in the equation that might be used to represent a tradeoff between different objectives. They can also choose the formulation of the objective function in another sense: an objective function may use, for example, a utilitarian formulation of compactness, representing compactness of the model in terms of the total compactness of the system;[39] compactness may instead, however, be represented using a Rawlsian approach, in which the goal would be to maximize the compactness

cartoonish real world example you might think of the multiplier as the fraction of space set aside for the dessert.

True Lagrangian multipliers would allow a computer to find the optimal amount of space to leave for desserts, so that it could maximize its objective function and meet its constraints: the fraction would not be fixed in advance. An automated redistricting model, however, might need to use weights instead of true Lagrangian multipliers. That is, the model designers would specify in advance the "portion of the tray" to be set aside for each goal, such as compactness and contiguity. The need to use weights in computer redistricting models has long been recognized. *See, e.g.*, Torricelli & Porter, *Toward the 1980 Census: The Reapportionment of New Jersey's Congressional Districts*, 7 RUTGERS J. COMPUTERS, TECH. & L. 135, 155 (1979).

37. In multiattribute annealing problems, these weighting choices are usually made empirically. *See, e.g.*, Koch, Marroquin & Yuille, *Analog "Neuronal" Networks in Early Vision*, 83 Proc. Nat'l. Acad. Sci. U.S. 4263, 4265 (1986) (finding weights in multiobjective vision application); Marroquin, *Surface Reconstruction Preserving Discontinuities* 11 (M.I.T. Artificial Intelligence Lab Memo No. 792, 1984) (using empirical testing to select values for parameters used in annealing application in image processing). Although the choices used may be made empirically in the redistricting case, this does not mean that the choices may be arbitrary.

38. This is similar to the Nagel model. *See* Nagel, *supra* note 7.

39. *See, e.g.*, objective function *supra* note 30. The measure used in that equation is that prescribed in Weaver & Hess, *supra* note 7, at 296-300.

of the least compact district.[40] Such a Rawlsian/utilitarian choice is present in the selection of other factors as well, such as population equality. Annealing has no consistency requirement mandating that the same type of formulation be made for each term. The model allows yet another choice in the measure of criteria chosen. For example, a formulation may use any one of a number of measures of compactness.[41] Still another choice may be made in the adjustment of touchlist values, if such changes are allowed.

If automation of the redistricting process should be a legislative function, then legislators will, in fact, have a chance to influence the results of the model through their choices. However, automating the process leaves the legislators accountable to both the public and the courts in their selections.[42] Because any chosen model may be subjected to court and public review, legislators will have an incentive to work with experts in redistricting to choose supportable terms and measures. For example, the modelers may have to defend the terms they used, such as the reasonableness of including or excluding a measure of compactness. The modelers might also have to defend their choice of a Rawlsian versus utilitarian measure of variables such as population equality or compactness.

Some observers may even scrutinize formulations of factors like compactness. For example, although the Court might simply want to see compactness in districting,[43] it is not clear that just any measure of compactness should be used by designers. It is possible to design compactness measures that are defensible from both a scientific standpoint[44] and a logical perspective.[45] Since compactness is a

40. In mathematical terms, that might be:

$$\min_{\text{districts } i} \ \max \sum_{\substack{\text{EDS} \\ j \in i}} \text{population}_j \,(\text{distance from center of } j \text{ to population center of } i)^2$$

41. Combinations of criteria could be used as well. One may, for example, design an objective function under which the solution would not measure poorly under any of several compactness measures, even though that configuration would not be a unique optimal under any single measure of compactness. The same steps could also be taken for criteria other than compactness.

42. *See generally* Note, *supra* note 3.

43. *See* Davis v. Bandemer, 478 U.S. at 167 (Powell, J., concurring in part and dissenting in part).

44. *See* description in Weaver & Hess, *supra* note 7, at 296-300.

45. For example, the Weaver & Hess measure, described at *id.*, makes sense largely because it maintains compactness as a relative rather than an absolute measure. Using the Weaver/Hess measure, one can say that District A is more compact than District B, but one cannot say that District A *is* compact. Unless a district consists of a single person in a single point, it logically does not make sense to be able to say that a district is compact in an absolute sense; thus any measures of absolute compactness should be suspect.

new area of research,[46] and it may be measured in many different forms,[47] any measure might be suspect unless it can be clearly defended by the modelers.

It is perhaps more likely that courts might simply require designers to show that the selections made were not purely arbitrary. However, even in that instance, legislators might hesitate before making particularly poor or "unfair" choices that could irritate the voting public, the courts, or their political rivals who have an incentive to scrutinize redistricting models and make their impressions known. While legislators might try to use proxies to conceal inappropriate choices, disclosure of the model design process succeeds in safeguarding the public to a large degree.[48]

The multipliers used to trade off the terms in the objective function will also affect the outcome of the redistricting process. Empirical research is needed to test the sensitivity of the terms;[49] for example, one might expect that as long as the multiplier of population equality lies in a certain range, the results should also lie in a range that meets court standards. Because an increase in the number of terms considered in the model will make it more difficult for an automated process to meet all goals,[50] it is reasonable to expect legislators first to create a model striving for only those features, such as population equality, contiguity, and compactness, that the Court has suggested, and later explore the use of additional, more political terms, such as those that would protect incumbents. Legislators must consider, though, that the use of a computer model in redistricting makes their choices highly visible. While modelers could, for example, include features in the program that would keep incumbents in separate districts, ostensibly at the expense of reaching some other goal, constituents might protest if they felt their political choices were being manipulated.

46. *See, e.g.*, Hofeller & Grofman, Comparing the Compactness of California Congressional Districts under Three Different Plans: 1980, 1982, and 1984 (1987) (unpublished manuscript); Niemi & Wilkerson, Compactness and the 1980s Districts in the Indiana State House: Evidence of Political Gerrymandering? (unpublished manuscript).

47. For example, measures include "the ratio of the district area to the area of the smallest circle which contains (circumscribes) the district;. . . the ratio of the district area to the area of the circle whose circumference is identical to the district perimeter . . . ," and "the ratio between the population of each district and the population of the area inside the polygon with the shortest possible perimeter length which completely surrounds the district." Hofeller & Grofman, *supra* note 46, at 3-4.

48. For a further discussion of these ideas, see generally Note, *supra* note 3.

49. *See* sources cited *supra* note 37.

50. *See supra* text accompanying note 18.

IV. Conclusion

Obviously, a great deal of research still must be done before implementing an automated redistricting scheme. However, the technique of simulated annealing is a promising solution to the redistricting problem. The development of a simulated annealing model for political redistricting could potentially eliminate the lengthy litigation and uncertainty now surrounding redistricting. An automated model, once developed, could be used every ten years to redistrict states quickly after each census. Moreover, having a fixed, legally acceptable method of automatic redistricting could eliminate the threat that the continuing crisis over redistricting poses to the legitimacy of our government.[51]

Unfortunately, it is impossible to eliminate some of the problems remaining in developing simulated annealing as an automated redistricting scheme. Most notably, choosing the weights for a multiattribute problem (the energy equation) may cause problems if weights can be chosen only empirically.[52] Also, battles over the structure of the program and the input data used could rival current lengthy court battles over political redistricting results. However, once an automated computer model is developed, and the challenges to its development have been addressed, that model may be used in subsequent redistricting efforts, avoiding the necessity of going through this battle every ten years. Once a model is developed, automated redistricting should prove very efficient in terms of human hours consumed.

It would be worthwhile for groups such as the Republican and Democratic National Committees and political action groups actively involved in redistricting to examine further the possibilities of computer modeling and simulated annealing, since a concentrated effort in this area might produce redistricting plans that could pass court review. The implementation of computerized redistricting could also ultimately save time and effort for legislators and judges, while improving the quality of political representation. As Holmes suggested years ago, "very likely it may be that with all the help that statistics and every modern appliance can bring us there never will

51. The present threat stems largely from concern over the extraordinarily high reelection rate of incumbents. *See, e.g.*, Rosenbaum, *It's a House of the Same Representatives*, N.Y. Times, Sept. 25, 1988, at E1, col. 1 (citing reelection rate as high as 98% in 1986). Although this reelection rate is partially attributable to campaign finances, one cannot eliminate redistricting as one of the causes of low turnover.

52. *See supra* note 37.

be a commonwealth in which science is everywhere supreme. But it is an ideal, and without ideals what is life worth?"[53]

53. Holmes, *Law in Science and Science in Law,* 12 HARV. L. REV. 443, 462 (1899).