

PUSHING THE ENVELOPE: ANTITRUST IMPLICATIONS OF THE ENVELOPE THEOREM

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I. INTRODUCTION

Aaron Director and Edward Levi discuss the difficult problems of regulating monopoly firms that “acquired their size without combination.”¹ In their thoughtful discussion of *Alcoa*,² Director and Levi follow the convolutions of Judge Learned Hand’s opinion in which Alcoa is ultimately culpable for anticipating the increases in the demand for ingot by “doubling and redoubling its capacity.”³ Commentators and courts have long struggled to distinguish impermissible from benign acquisition of monopoly power.

In these remarks, I would like to argue that our antitrust law has paid too much attention to how monopolists acquired their monopoly power and not enough attention to whether a monopolist has exploited its monopoly power by raising prices. While monopoly power might be thrust upon a firm, supra-competitive pricing is not. My thesis is that liability for monopolization should turn more on evidence about the *degree* of the monopoly overcharge. In particular, we should structure our antitrust laws to deter monopolists from charging the full monopoly price.

This thesis is a fairly straightforward implication of the “envelope” theorem. The first section will describe the “envelope” theorem, and the second section will then develop its implications for antitrust law.

II. THE ENVELOPE THEOREM

The envelope theorem has primarily been used by economists as a tool to solve mathematical models⁴ — and almost never appears in law review articles.⁵ However, George Akerlof and Janet Yellen published a classic article ten years ago drawing implications of the envelope theorem for a wide variety of contexts.⁶ My comments here extend their insights to antitrust.

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1. Aaron Director & Edward H. Levi, *Law and the Future: Trade Regulation*, 51 NW. U. L. REV. 281, 284 (1956), reprinted in 17 MISS. C. L. REV. 7, 10 (1996).

2. *United States v. Aluminum Co. of Am.*, 148 F.2d 416, 431 (2d Cir. 1945).

3. Director & Levi, *supra* note 1, at 286 (quoting *United States v. Aluminum Co. of Am.*, 148 F.2d 416, 431 (2d. Cir. 1945)).

4. Formally, the envelope theorem is the result of a maximization problem: Consider a decision maker maximizing the function $f(x, a)$, where x is the decisionmaker’s choice variable and a is a parameter exogenous to the decisionmaker. Let $x(a)$ denote the unique maximizing choice of x , given a , and let $M(a) = f(x(a), a)$ denote the maximum value of f for given a . According to the envelope theorem: $dM(a)/da = \partial f(x(a), a)/\partial a$. Given that the decision maker is choosing x to maximize $f(x, a)$, the total derivative of the function with respect to a is equal to the partial derivative of the function. See HAL R. VARIAN, MICROECONOMIC ANALYSIS 267, 268 (1978).

5. A Lexis Search uncovers only two references in law reviews. See Thomas G. Krattenmaker & Steven C. Salop, *Anticompetitive Exclusion: Raising Rivals’ Costs To Achieve Power Over Price*, 96 YALE L.J. 209 (1986); Eric L. Talley, *Contract Renegotiation, Mechanism Design, and the Liquidated Damages Rules*, 46 STAN. L. REV. 1195 (1994).

6. George Akerlof & Janet Yellen, *Can Small Deviations From Rationality Make Significant Differences To Economic Equilibria?*, 75 AM. ECON. REV. 708 (1985).

Let me begin with a simple test. Consider a monopolist with constant marginal costs facing a linear forty-five degree demand curve. Using the monopoly price as a benchmark, what would be the impact on the monopolist's profits and on the deadweight loss if we could induce this monopolist to cut its markup by ten percent? A ten percent reduction in its markup reduces monopoly profit *by less than one percent*, but reduces the deadweight loss of monopoly by almost twenty percent. Even though antitrust teachers draw these types of demand and cost curves on the board countless times in the course of a semester, my intuition is that few of us are aware that the last bit of the monopoly price hurts society much more than it helps the monopolist.

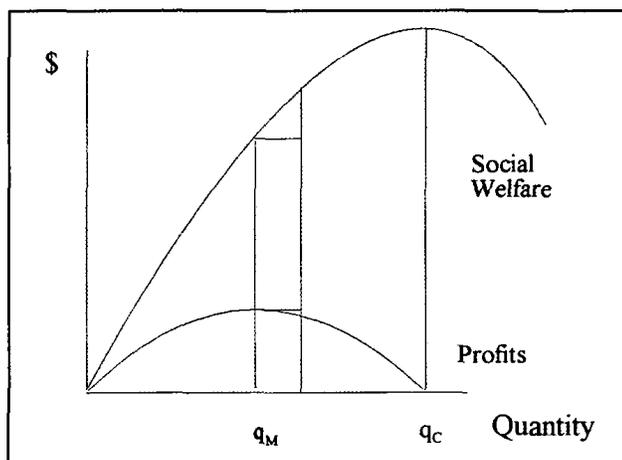


Figure 1: Increasing Monopoly Quantity (q_{monop}) Has First-Order Effect on Social Welfare But Only Second-Order Effect on Profits

This envelope theorem explains why this is so. Because the maximum point on a sufficiently smooth curve is flat, small changes from the maximum will only have a small effect on the height of the curve. As shown in Figure 1, because the monopolist's profit curve is horizontal at the monopoly quantity, a ten percent markup decrease (and attendant increase in quantity) does not greatly reduce the monopolist's profits. The social welfare curve, however, is not horizontal at the monopoly quantity so that the ten percent decrease in markup has a much larger impact on social welfare. Because the monopolist pushes output to the flat or horizontal part of the "envelope," small movements away from this point have only second-order effects on profits.

The intuition behind the envelope theorem can also be seen in the more traditional monopoly pricing diagram. As shown in Figure 2, a reduction in the monopoly price from P_M to P' has two effects on the monopolist's profits: the lower price reduces the profits on current sales (the reduction in profits is represented by the area of rectangle A), but the lower price also increases the quantity sold (from q_M to q') at a supra-competitive price (this increase in profits is represented by the area of rectangle B). So geometrically, the effect on profits of lowering the price below the monopoly level is:

$$\text{Effect on Profits} = \text{Area B} - \text{Area A}.$$

At the profit-maximizing price, these two effects on profits offset each other, but even reductions in price below the monopoly price cause a relatively small decrease in profits because the increase in profits from additional sales (Area B) almost offsets the decrease in profits from a lower price (Area A).

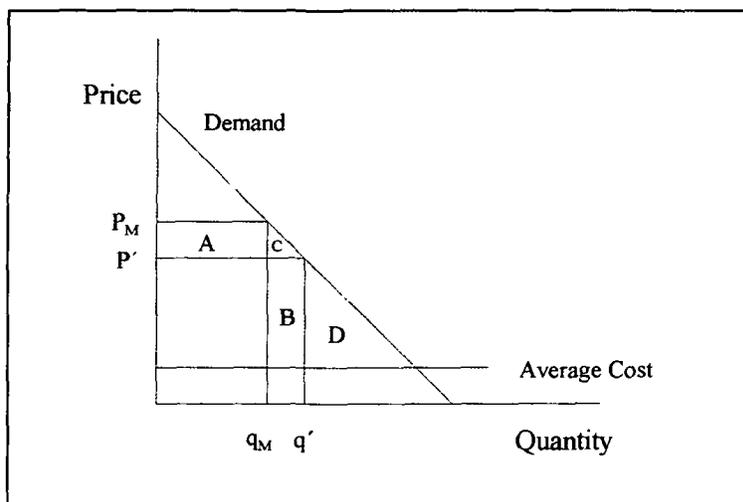


Figure 2: Graphic representation of price decrease on profits and deadweight loss.

This reduction in profits is especially small in comparison with the increase in social welfare (consumer and producer surplus). As shown geometrically in Figure 2, the increase in welfare caused by lowering the monopoly price is:

$$\text{Effect on Welfare} = \text{Area B} + \text{Area C}.$$

It is well known that the monopolist's loss of Area A profits does not reduce social welfare because what the monopolist loses in this area, the consumer gains. But what is not often emphasized is that the effect on profits is much smaller than the effect on social welfare. Because of the offsetting profit rectangles, the effect on profits is *second-order* in comparison with the *first-order* effect on social welfare.⁷

As a mathematical theorem, the envelope result only holds "locally" — that is for arbitrarily small changes from the profit maximizing point. As Akerlof and Yellen have observed: "But for the theorem to have practical relevance, it must be true for finite [changes]."⁸ But as our linear example showed, price reductions which reduce twenty percent of the deadweight loss of monopolization might only induce a trivial reduction in monopoly profits. This result is not a patholog-

7. The envelope theorem also has implications for thinking about increasing the price above the competitive level. The social welfare function is flat at the competitive price, so that a small increase in price above the competitive level will reduce only a small amount, but will increase monopoly profits substantially.

8. Akerlof & Yellen, *supra* note 6, at 711.

ical artifact of assuming a linear demand curve or constant marginal costs. For a wide array of cost and demand curves, substantial improvements in social welfare would result from price reductions which only trivially reduce a monopolist's profits.

III. IMPLICATIONS FOR ANTITRUST LAW

The central implication of the envelope theorem for antitrust law is that it should not take very large carrots or sticks to induce a monopolist to reduce its price below the profit-maximizing price. Director and Levi knew that the profit-maximizing price will always be a function of expected legal liability: "In the case of the assured monopoly, one may predict a restriction on production because this restriction will be sensible from the standpoint of the firm. *To be sure, even then the firm will wish to take into account . . . the threat of governmental intervention.*"⁹ But what these authors did not see was that a monopolist would need very little governmental inducement to cut its markup ten percent.

Appreciating the envelope theorem, the Justice Department might want to target firms that seem to be charging too much. The government's recurring scrutiny of Microsoft might be justified even if it created only a small probability of ultimate liability. But in pursuing such a strategy, the government should state that it would be less likely to prosecute (or would call off investigations) if firms restrained themselves from fully exploiting their monopoly power.

We might even want to go as far as to allow a reasonable price defense. In the absence of legal incentives, a monopolist always has an incentive to raise price until the demand becomes elastic. Accordingly, evidence that a monopolist was producing on an inelastic portion of its demand curve would be strong evidence that the monopolist was setting price below the profit maximizing level.

Finally, we might want to modify treble damages as the across-the-board award for competitive overcharge. The envelope theorem suggests that society is hurt much more by the last increment of monopoly overcharge than the first increment. In keeping with the conventional notion that damages should be set so that the injurer "internalizes" external costs, it makes sense to have progressive punitive damage multipliers that more closely approximate the social costs.

The most serious problem with implementing any of these proposals is that government actors would have great difficulty estimating the degree of a monopolist's overcharge. Just as it has proven difficult in predatory pricing cases to discern whether a firm is pricing below the competitive price, it would be difficult to estimate whether a firm was pricing below the monopoly price. But even if the court estimates the monopoly price with error, I hope to show in a later project that a legal system that was attuned to the degree of the monopoly overcharge could outperform the current system. In the end, we might still conclude that the envelope-inspired policies are ill advised because of problems with estimation. But at the very least, we should recognize that the goal of deterring what

9. Director & Levi, *supra* note 1, at 287 (emphasis added).

I have called “the last bit” of the monopoly overcharge is particularly appealing because it hurts society much more than it helps the monopolist.

A less serious reason to avoid making liability turn on the degree of the monopoly overcharge might be a belief that very few firms price at the monopoly level. Monopolists might already be deterred from charging full freight if they were trying to (a) deter potential competitors from entering; (b) exploit a learning curve; or (c) build a network of loyal consumers. But the argument that few monopolists engage in full monopoly pricing would suggest that the envelope-inspired policies would not affect many firms, not that the policies themselves are wrong-headed.

IV. CONCLUSION

In their first year, law students quickly learn that the contract law defense of “unconscionability” almost never is used to regulate the price term of contracts. Some professors mollify students by suggesting that unconscionably high prices are a problem of market failure and therefore a concern of antitrust law. But surprisingly, the antitrust law of monopolization also refuses to directly scrutinize the reasonableness of the monopolist’s price. Supra-competitive price is *the* evil consequence of monopolization, but our antitrust laws are curiously indifferent to its existence — liability does not turn on the existence of supra-competitive pricing.

A central implication of “marginalism” is that society should consider the marginal costs and benefits of particular activities. For example, in the pollution context, we might not want to completely clean up all of an oil spill, even if the total benefits are greater than the total costs, because cleaning up the last few gallons may cost more than the marginal benefit. But my thesis has been that our antitrust policy has not been attuned to “marginalism.” Each succeeding dollar of a monopolist’s overcharge is not equally harmful; the last bit is disproportionately harmful.

Because economic analysis has come to dominate antitrust, it is especially bizarre that marginal deterrence does not play a larger role in the policy debate. The central implication of the envelope theorem is that it would take very little to deter monopolists from exploiting the last bit of their monopoly power, but far from harnessing this powerful ally, current rules give a monopolist no incentive to restrain its pricing. Trying to deter that last bit of monopoly pricing is especially appropriate because courts have trouble discerning whether monopoly power is legitimately acquired or not. The envelope-inspired policies are “second-best,” in that it would be preferable to deter all monopoly power. But in a world where we cannot or do not want to deter all monopolies, we may still want to deter those that do exist from producing the most costly social externalities.

