Among the more significant terms in this article there are two of paramount importance—Mathematics and Science. Rightly understood they are probably the weightiest terms in the vocabulary of rational thought. But these great terms, though spoken or written by all, though heard and seen on every hand, are seldom employed with anything like a critical understanding of their respective significations. In common parlance they are never so employed; and it must be said that, with rare exceptions, eminent scholars themselves, even in their gravest discourses, employ them with but slight regard to precision and clarity of meaning. Clarity is no doubt the highest stylistic obligation of an author to a reader whose strongest desire is the desire to understand. Clarity, however, is not easy to achieve; and in an essay dealing succinctly with a vast and intricate subject one may not reasonably hope that all of one's statements are quite without ambiguity or indetermination. Yet it must be held to be the minimal requirement of such an essay that its major theses shall be so presented that no competent and attentive reader need remain in doubt as to what it is that such theses assert. In order to meet that requirement in the present case it is necessary to indicate in advance, as clearly as may be done in a few words, the respective senses in which the term mathematics, the term science, and various derivative terms are to be employed throughout the discussion.

To indicate the sense in which the term mathematics is to be understood I have only to report what centuries of criticism have at length shown us that mathematics essentially and distinctively is. But for the term science my task is entirely different. For this term has never been defined and so there is no existing definition of it to report. In saying this I assume that the reader is well aware of the radical difference between definition and
description. In what is called scientific literature there are to be found many more or less apt partial _descriptions_ of what the writers have variously supposed the term science to represent, but neither in that literature nor elsewhere have I been able to find anything that may, properly speaking, be called a _definition_ of science. In order to indicate clearly the sense in which the term is to be understood in this essay, I am, therefore, obliged to construct, instead of merely reporting, a definition of the term. It will be found, I believe, that the definition which I intend to submit, not only does not contravene anything of essential importance in the descriptions above alluded to, but has, moreover, the immense advantage of assigning to the term science a significance as definite as that of mathematics or of any other term in speech. The reader may find it worth noting that neither the reported definition of mathematics nor the proposed definition of science is methodological but that both of the definitions are based upon the kinds of propositions—the two natural propositional types—with which, it is held, mathematics and science (rightly conceived) respectively deal.

The function of propositions is to answer questions. Questions are of two kinds: questions regarding the make-up of the actual world and questions regarding the make-up of the world of possibility. The former kind may be simply exemplified by such questions as these: What is the specific gravity of iron? What are the essential functions of government? What is the shape of the earth? What is the social value or effect of the human sense of obligation? What is the velocity of light? Questions regarding the world of the possible may be simply exemplified by such as these: If the geometric axioms of Lobachevski were valid, what other propositions would then necessarily be valid? If there were a 4-dimensional space related to ordinary (Euclidean) space as the latter is related to an ordinary plane, what would be the geometric structure of that 4-dimensional world? If John Doe were in Chicago, at the time when it is alleged, he stabbed Richard Roe in New York, what other relevant propositions or propositions would of necessity be true?

Corresponding to the two great question-types, which together embrace all possible questions, there are two natural types of propositions, which together embrace all possible propositions: _categorical_ propositions, saying that such-and-such is the case, in response to questions relating to the actual world; and _hypothetical_ propositions, which say that, if such-and-such supposable things were actual, then of logical necessity such-and-such other things would be so too, in response to questions about the possible world.

More precisely, if \( p \) and \( q \) be propositions, then the proposition, \( p \) implies \( q \), meaning that \( q \) is logically deducible from \( p \), is hypo-
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Theoretical; all other propositions are categorical. It is common and convenient, though often misleading, to state hypothetical propositions in the form: if \( p \), then \( q \); misleading because categorical propositions are often put in that form. For example, the proposition that the orbit of the earth is an ellipse is categorical and it does not cease to be such when stated in the *if-then* form: if \( O \) be the orbit of the earth, then \( O \) is an ellipse. Again, the proposition—If Doe strikes Roe, then Roe will get angry—is categorical; it does not mean to assert that “Doe strikes Roe” implies “Roe will get angry;” in other words, the proposition does not intend to assert logical deducibility. The form, \( p \) implies \( q \), should be taken as the *standard* form of hypothetical propositions, \( p \) being the hypothesis or implier and \( q \) the consequence or implicate.

Mathematics is the enterprise having for its aim to establish hypothetical propositions.

Science is the enterprise having for its aim to establish categorical propositions.

The two enterprises together embrace the whole knowledge-seeking activity of man.

A mathematical proposition is an established hypothetical proposition.

A scientific proposition is an established categorical proposition.

By the phrase, an established proposition, is meant a proposition that is so spoken of, so regarded, so treated by all or nearly all authorities in the subject or field to which the proposition belongs. It is evident that an established proposition has a date or dates, for a proposition may, during a period of time, be an established proposition and then cease to be such. I need not give examples.

Mathematical method consists of all the means available for establishing hypothetical propositions, logical deduction being the ultimate test in every case.

Scientific method consists of all the means available for establishing categorical propositions, observation being the ultimate test in every case.

**FORM AND CONTENT**

It is essential to note and to keep in mind that a mathematical proposition has, strictly speaking, no content, or subject matter. For if \( p \) implies \( q \), the deducibility of \( q \) from \( p \) depends only on the *forms* of \( p \) and \( q \) and not at all on their content (if they happen to have any) and, as already said, it is such deducibility and nothing else that a mathematical proposition asserts. On the other hand, a scientific proposition always has content, or subject-matter, for it is a categorical proposition purporting to an-
swer a question respecting some portion or feature of the actual world. It would be difficult to exaggerate the critical importance of the distinction signalized in the last three sentences.

THE SUBJECT-MATTER OF LEGAL SCIENCE

Such distinctions as I have now very briefly, perhaps too briefly and imperfectly, indicated, are essential to the meaning of the following remarks regarding the study of legal science. Every branch of science has a subject-matter of its own—some group of natural phenomena—some fragment or aspect of the actual world. If the study of law is, or is to be, the study of a branch of science, then it must deal with a group of natural phenomena, with a part of the actual world, with some peculiar type of subject-matter.

What is that subject-matter?

In my present view the answer is: the subject-matter of legal science is a certain species of human behavior—I mean the distinctive behavior of those persons whose official rôle in human society is to answer, for the community they represent, such questions as arise respecting what is just. In a word, the subject-matter of legal science is the decisions (the distinctive behavior) of judges. In making these statements I am not unaware of the ambiguity attaching to the terms, “decision” and “just.” I trust, however, that the meaning of the statements, interpreted contextually, will be sufficiently clear for the purpose they are to serve.

The subject-matter indicated is a perfectly natural subject-matter. It is a genuine part of the natural world. It is something to be observed and studied like any other part. Human beings are literally a part of nature. It is essential to realize that a society or community of our human kind is just as natural as the ground they occupy, as the air they breathe, as the light and heat of their great fire-side, the Sun. The many-colored stream of human life is literally a part of the cosmic stream. Looked upon objectively, the stream of human life presents many sorts of phenomena—some for the psychologist, some for the moralist, some for the religionist or theologian, some for the physician, some for the political scientist, and so on. Among the various types of natural phenomena observable in the flowing spectacle of the life of human society, there is present also a most notable group of phenomena constituting the jural type—the distinctive acts or deeds or performances or decisions of judges—the behavior of the judiciary as such. That behavior

1For a full discussion of these and kindred distinctions the reader may be referred to my THE PASTURES OF WONDER: THE REALM OF MATHEMATICS AND SCIENCE (1929).
is a genuine part of the life of mankind; it is, like human beings themselves, as natural as anything else in the natural world; it is, if you wish so to view it, literally a part of cosmic behavior; and it is, I submit, the subject-matter of legal science.

WHAT WILL LEGAL SCIENCE CONSIST OF?

A branch of science may be viewed as an enterprise or as a body of propositions. From what has been said it appears that, regarded as a body of propositions, the science of law will consist of categorical propositions setting forth the distinctive behavior of the judiciary together, of course, with the stimuli calling it forth and the circumstances conditioning it.

Regarded as an enterprise legal science will have for its aim the establishment of such categorical propositions.

NOT TO BE CONFUSED WITH THE DOCTRINE OF STARE DECISIS

Is what I have been saying just an ignorant layman's crude attempt to state the ordinary conventional conception of stare decisis? I am quite prepared to admit the ignorance and the crudeness but am far from being prepared to admit that the usual concept of stare decisis is equivalent to the concept I have been trying to set forth.

BRANCHES OF LEGAL SCIENCE

Thus far, I have been speaking of a science of law in general. It is, of course, obvious that legal science will have numerous branches. There will be a science of the law of contracts, a science of the law of agency, a science of the law of torts, and so on and on. Each of these will have for its subject-matter a part of juridical behavior. And it is plain that each of the indicated branches of legal science will have sub-branches according as the inquiry relates to this, that, or another community or state, it being understood that two communities may exist at the same time in different places or at different times in the same place.

EMERGENCE OF A GREAT PROBLEM

It seems evident that a great problem—a problem of comparative law—would be that of ascertaining: (1) what, if any, propositions are common to the various time-and-place subdivisions of a given branch (say that of contracts); (2) what, if any, propositions are common to all branches of a given time or place; (3) what, if any, propositions are common to all branches of all times and places.
THE QUESTION OF PREDICTION

It is a familiar and just saying that a test of a science is that a genuine science makes it possible to predict. An attempt to predict based on legal science or a branch thereof employs the assumption or postulate that a given pattern of judicial behavior will repeat itself when the corresponding stimuli and conditioning circumstances repeat themselves. Is the postulate sound?

It is a fact that stimuli and circumstances seldom or never do repeat themselves exactly. That fact being sufficient to explain why exact prediction of judicial behavior is impossible, we need not deny the validity of the postulate upon which attempted prediction depends.

LAW IS A VARIANT FUNCTION OF VARIABLES

The fact is obvious that the cosmic stream including the stream of human life goes flowing on; the new is ever emerging without exact repetition of the old; and, inasmuch as judicial behavior is a part of the life of mankind, we should expect, what we see to be a fact, that law is not an invariant somewhat but is a variable, changing with time and place and the things that these involve.

Law (judicial behavior) changes because the stimuli that evoke it and the circumstances that condition it do not remain the same and do not repeat precisely but continually alter under the influence of new things emerging endlessly in the flux of life and the world.

We are thus led to employ the scientific notion of functionality in the study of legal science. It is a well known fact of observation that law (judicial behavior) depends upon and varies with a variety of more or less familiar variables. And so in functional notation we may write

\[ L(v_1, v_2, v_3, \ldots v_n) \]

merely to indicate the thought that the law \( L \) is a function of certain variables \( v_1, v_2, \ldots \) and undergoes changes due to changes in them.

Immediately certain queries and facts stare us in the face.

(1) What are the variables \( v \)? As a very rough suggestion we might assign meanings to the \( v \)'s as follows:

- \( v_1 \) stands for modes and forms of business, 
- \( v_2 \) stands for manners, customs, mores, 
- \( v_3 \) stands for religious opinion and feeling, 
- \( v_4 \) stands for science and invention, 
- \( v_5 \) stands for industrial development, 
- \( v_6 \) stands for political theory, 
- \( v_7 \) stands for axiology, 
and so on.
If we give the \( v \)'s some such meanings as those indicated, we immediately see that, though the variables are distinct, they are not independent of each other. In that case our function \( L \) is a function of numerous variables each of which is at the same time a function of all the others. Thus arise new problems of functional relationship—complications that students of legal science have to face.

Is it possible to find or select a set of variables \( v \) such that they shall be at once independent and exhaustive? Is it probable that such a set exists?

If we make any reasonable specification of what the \( v \)'s are to represent, then, though we know that \( L \) depends upon them, we do not know how it depends upon them; that is to say, we do not know in advance, as we often do in mathematics, the form of our function \( L \). On the contrary, the form of \( L \) remains to be discovered, if possible. But that fact is to be expected, for scientific functions, unlike many mathematical functions, are not given in advance. I mean that their forms are not known in advance but require to be ascertained by observation of the facts to which they relate.

Given a set of \( v \)'s, can the form of \( L \) be ascertained? It is almost certain that it cannot by any known means or in any time imaginable but that is not a good reason for failing or refusing to envisage the problem.

STATUTORY LAW VIEWED AS JUDICIAL BEHAVIOR

I have said that the subject-matter of legal science is the distinctive behavior of judges and that the science of law must consist essentially of categorical propositions setting forth such behavior together with the evoking stimuli and the conditioning circumstances. It is common to say, however, that not all law is judge-made, that what is called statutory law is not judge-made but is made by legislation. The distinction seems very obvious at first. Will it bear close scrutiny? It may be doubted. It appears to me that a statute (a verbal formula set up by a legislature) is not itself a law but that, rightly conceived, a statutory law is in fact the judicial interpretation of a statute. That seems indisputable in the case of a statute so ill-drawn that its meaning is obscure. Let us suppose, however, that a statute is so clear that even an illiterate person, upon hearing it read, would instantly understand it. Nevertheless a court may, if it will, assign to the statute a meaning obviously inconsistent with the meaning it was evidently designed to convey. In that case, if the court be one of the last resort, the assigned meaning, the judicial interpretation, is, until reversed or altered, statutory law. On the other hand, if the meaning declared by the court coincide, as it normally will, with the evident meaning of the
statute, such declared meaning is none the less an expressed judicial interpretation, and, again, this interpretation is a statutory law. The statute itself is to be rightly viewed, I believe, as one, though it is often the controlling one, of the circumstances conditioning judicial behavior. Because that circumstance, when it is present, is so important, students of legal science are confronted with a special problem. I mean the problem of rightly conceiving the relation between legislation and judicial determination—between statute and law.

THE RÔLE OF MATHEMATICAL METHOD
IN THE STUDY OF LEGAL SCIENCE

In studying judicial behavior it is very noteworthy that in all or nearly all important cases the statement of a decision occurs as part of a discourse purporting to set forth the principles and processes by which the decision is arrived at and is held to be justified. Such discourses constitute an immense literature, ever growing. That literature presents certain very striking phenomena to which students of legal science must necessarily give a great deal of serious attention. These phenomena are of four kinds or types as follows:

(1) Terms employed deliberately or unconsciously without being defined. Such terms may be called primitive terms. Primitive terms not only do occur but must occur, for no discourse can define all of its terms without committing the unpardonable sin of circularity.

(2) Numerous defined terms—defined, of course, by help of the primitives. Definitions are not logical necessities but they are economic ones.

(3) Propositions consciously or unconsciously taken for granted, assumed without attempted proof. They may be called primitive propositions or postulates. Postulates must occur, for no discourse can, without self-stultification, undertake or pretend to prove all of its propositions.

(4) Numerous “proved” propositions—ostensibly deduced logically from postulates.

Accordingly it seems evident that any one of the branches of legal science must confront the student with the following eight problems:

(1) To detect and list the terms actually employed (in the given branch) as primitives.

(2) To detect and list the terms ostensibly defined and to examine the definitions with reference to their clarity, precision and general availability.
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(3) To detect and list the propositions actually employed as primitive propositions, or postulates.

(4) To detect and list the propositions ostensibly proved or deduced and to scrutinize the proofs for their logical soundness or unsoundness.

(5) To determine what terms it would be most expedient to select for use as primitives.

(6) To construct the most available definitions of all other important terms by means of the primitives.

(7) To determine what propositions it would be most expedient to select as primitive propositions, or postulates.

(8) To deduce all other propositions logically from the chosen postulates.

Not only do the eight problems require to be solved for each branch of legal science but—what is vastly more—they require to be stated and solved for legal science as a whole.

WHAT IS THE USE OF MATHEMATICIZING?

It is natural and pertinent to ask what good can come from mathematicizing a branch of legal science. As a brief general answer to that question I would submit the following three fairly obvious considerations:

(1) The mathematical method can never tell what the law is or ought to be. It can tell what the law is or ought to be only upon the assumption that certain postulates have been granted. Nothing, however, but experience can ultimately tell whether a given system of postulates ought to be granted, ought, I mean, in the interest of justice. But mathematics can help experience in this matter, for it is precisely the peculiar office of mathematics to deduce the consequences of a postulate system, and it is perfectly evident that such a pre-experiential knowledge of a postulate system's consequences must greatly facilitate the task of experience in judging the merits of the system itself.

(2) The method in question converts any legal branch to which it may be applied into an autonomous system, or doctrine, in which the component propositions are logically ordered and related. Such a system answers to a deep natural craving of the human intellect—a craving that reaches at once for the convenient, for the economical, and for the aesthetic.

(3) An ensemble of experience-given propositions (like those constituting any existing branch of law) never gets so thoroughly examined and criticized and understood as when the ensemble is submitted to the severe processes of mathematicization.
On the other hand, it seems probable that, if a legal branch were reduced to a perfectly logical system, the very perfection itself would make it somewhat harder to modify the system when the interests of justice required such modification. But that contingent disadvantage does not seem so very weighty compared with the advantages mentioned.

In view of the above-indicated dependence of law upon extralegal variables, it is evident that the study of legal science must reach far into many another field: history, scientific method, mathematical method, psychology, sociology, axiology, ethics, religion, political science, economic theory, industrial development, and so on.

In this essay I have refrained from using the phrase "philosophy" of law, because in my view present-day philosophy is not something distinct from science but is literally a genuine part thereof, as explained in The Pastures of Wonder, above alluded to.