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MODEL OR MUDDLE? Quantitative Modeling and the Façade of “Modernization” in Law

Lea Brilmayer* and Yunsieg P. Kim**

I. INTRODUCTION

Once considered fairly exotic, the appearance of mathematical formulae in legal cases and scholarly literature has now become relatively commonplace. One would not have predicted this development from the phenomenon’s humble beginnings. The earliest attempts at quantification were nothing more than simple calls to compile more accurate statistics. Such efforts led the Harvard Law Review in 1930 to quote with apparent approval a book editor, who lamented the “absence of current criminal statistics” and argued that “[t]he time is rapidly approaching... when... every specialist in educational research must at the same time be a statistician.”

Legal academics have since moved beyond arguing for the collection of more accurate quantitative data, calling for such things as more sophisticated uses of statistical methods, the use of statistical methods to determine causation in torts, and the use of quantitative models to assess future risk. In a discipline that was once shaped more by “methods of philosophy” than by the quantitative sciences, terms such as “the null hypothesis” and “statistically significant” have become familiar terms of art. But it is not only statistical methods that have been urged

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upon American lawyers; approaches based on economics and game theory have also been proposed. We examine the proposed applications of all three of these types of quantitative reasoning to law.

II. QUANTITATIVE METHODS IN LAW: THREE EXAMPLES

In the last few decades or so, the trend towards quantification has crystallized into several movements purporting to model important developments in real life. The first of these is "law and economics" (the modeling of private and sometimes public law by reference to economic principles) and the second involves conforming intuitive probabilistic reasoning to Bayes' Theorem. These two will be described and evaluated somewhat briefly, while a third invites assessment at greater depth. This third movement claims to model strategic decision-making through applications of game theory, purporting to support neoconservative and liberal political reasoning about whether international law has a legitimate claim to the status of "law."

A. Law and Economics: A Brief Sketch

Law and economics is probably both the oldest and the most successful of these quantification movements. The application of economic theories and models to legal theory is said to have begun in earnest in antitrust law, where economic concepts such as monopolization, price-fixing, and collusion have played a starring role ever since the discipline's inception. However, since the early 1960s, the application of economic concepts has spread to many other topics, from risk distribution in torts and the design of interpretive rules for the Internal Revenue Code to international trade regulations. Even as the application of economics to the study of law becomes fairly routine, law and economics advocates have continued their proselytizing, calling for ever more ambitious extensions of the economic approach to areas as diverse as legal history and the design of a regulatory regime to combat cli-
mate change.  

Law and economics proponents argue that their method yields more accurate legal analysis, with some scholars stating that the economic approach is "more scientific" and that the discipline of law, on its own, is not scientific. Importantly, the economic approach often incorporates rational choice theory, which assumes that people act in order to maximize their own private gain. For example, the economic approach better explains the behavior of judges than does the "orthodox notion that judges merely interpret and apply law," it is said, because the assumption of self-interest takes into account factors such as judges' desire for promotion.

Not all scholars buy into this methodology; some have criticized (in particular) the notion that law and economics is more precise than non-economic methods of legal analysis. There are those who argue that assumptions central to law and economics, such as the premise that everyone is a rational maximizer of self-interest, are inaccurate and that competing law and economics explanations are no better. Still others assert that the economic approach tends to distort legal notions grounded in non-economic concepts by forcing them into a narrative that is foreign to them, pointing out that "[r]eplacing one set of terms with another does not make these 'law and economics' discussions any more 'scientific' than conventional legal treatments."

B. Bayesian Probability Theory

Probability theory, possibly even more so than law and economics, aspires to usurp the place of traditional intuitive legal reasoning. Perhaps for this reason, it has captured the imagination of few practicing

14. ROBERT ELLICKSON, ORDER WITHOUT LAW: HOW NEIGHBORS SETTLE DISPUTES 147 (1991) ("Anyone who widely reads in both law-and-economics and law-and-society literature is bound to come away feeling that economists . . . are clearer, more scientific, and more successful in building on prior work.").


lawyers or judges. There are, however, other explanations for this relative obscurity: unlike law and economics, probability theory is not based on a way of approaching problems that is familiar to the educated public. The ambitious scope of law and economics probably reflects in part the simplicity of basic economic concepts, which lend themselves to a wide variety of applications that even a person relatively unfamiliar with technical fields can grasp—if not necessarily agree with.

Although there were occasional attempts to introduce probabilistic reasoning to the law of evidence as early as the mid-1950s, the modern era of Bayesian probability theory was inaugurated by a 1968 California robbery prosecution, *People v. Collins.*\(^{21}\) In that case, the judge allowed a prosecution expert to produce evidence intended to establish that the two defendants were actually the two persons who committed the robbery.\(^{22}\) The expert’s argument was that it was very unlikely that another couple in Los Angeles would share so many characteristics with the accused couple: the defendants were an interracial couple—an African American man with a mustache and a beard and a blond woman with a ponytail—and they were driving together in a yellow car.\(^ {23}\)

The expert purported to calculate the probability that the crime was committed by a different couple by multiplying the individual probabilities of the individual characteristics.\(^ {24}\) According to the expert’s reasoning, since the probability that the woman was blond was allegedly 1/3 and the probability that she had a ponytail was 1/10, the probability that both would be true was only 1/30.\(^ {25}\) Accounting for all of the characteristics in this way, the prosecutor opined that there was only one chance in twelve million that a couple selected at random would possess the incriminating characteristics.\(^ {26}\) The jury convicted.\(^ {27}\)

The California Supreme Court, unpersuaded, reversed.\(^ {28}\) The grounds for reversal largely reflected the technical deficiencies in the expert’s argument—one being that no evidence had been offered to show that the individual characteristics of the accused couple occurred independently from one another.\(^ {29}\) For example, a man with a beard may be more likely to also have a mustache; thus, one cannot simply multiply 1/4 (the alleged probability that a man would have a mustache)
by 1/10 (the alleged probability that an African American man would have a beard). 30 This point would be obvious to anyone versed in modern probability theory. Indeed, the technical errors with the expert's testimony were so egregious that it is baffling how the evidence was ever considered in the first place. 31 The prosecution's characterization of the testimony as "expert" surely helps explain why this blatantly erroneous argument was persuasive to the jury.

The Collins fiasco sparked efforts by a lawyer/statistician team to salvage the idea of applying probability theory at trial. In 1970, the Harvard Law Review published A Bayesian Approach to Identification Evidence, co-authored by lawyer Michael Finkelstein and the now-eminent statistician William Fairley. Their article exposes the numerous methodological errors in Collins and proposes an alternative quantitative methodology based on the 1763 writings of Thomas Bayes. 32 Finkelstein and Fairley explained the need (as they saw it) for employing Bayes' Theorem in criminal trials as follows:

To test the utility of the explicit use of Bayes' theorem, the authors conducted an informal survey of intuition by using the facts in the case of the murdered woman . . . . The subjects . . . were first given the facts, excluding the palm print information, and asked to assess the probability of defendant's guilt. They were then given the palm print statistics and asked for a reassessment . . . . In almost all cases the addition of the palm print evidence was thought to raise the probability of guilt . . . . In most cases the assessments were not as great as they would have been if the probabilities had been computed in accordance with Bayes' Theorem. 33

They argued, in short, that Bayes' Theorem is needed because not using Bayes' Theorem results in lower estimates of the likelihood that the couple before the court is guilty. Application of Bayes' Theorem was necessary, according to this argument, to obtain correct results.

With Finkelstein and Fairley essentially claiming that Bayes' Theorem is useful simply because intuitive reasoning without the theorem would yield different results, it should not be surprising that their proposal has not, thus far, been accepted. They never explain why (in their

30. Id. at 38.
31. See id. at 40. For example, as the Court pointed out, this approach attempted to quantify things that cannot be quantified. See id. ("[T]he likelihood of human error or of falsification obviously cannot be quantified. . . .").
32. Michael O. Finkelstein & William B. Fairley, A Bayesian Approach to Identification Evidence, 83 Harv. L. Rev. 489, 516 (1970). Bayes' Theorem calculates the probability of an event occurring, assuming that another event will definitely occur. Assuming that P(A|B) denotes the probability of event A given the occurrence of event B, and P(A) denotes the probability that event A will occur, Bayes' Theorem is commonly stated as: P(A|B) = \frac{P(B|A)P(A)}{P(B)}. For a more detailed mathematical proof of Bayes' Theorem, see Arnold Zellner, Statistics, Econometrics, and Forecasting 8 (2004).
33. Finkelstein & Fairley, supra note 32, at 502 n.33 (emphasis added).
view) the Bayesian method is actually "better"—other than, perhaps, by insinuating that the application of a mathematical theorem must always be superior. This is the mathematical equivalent of "he said, she said." The reader is not informed of the controversy that surrounds such applications of the theorem, even in the probability literature.\textsuperscript{34} Interestingly, in areas of law where statistical methods have been employed (such as Title VII discrimination cases) the type of reasoning used is typically not Bayesian.\textsuperscript{35}

Since Finkelstein and Fairley's article, periodic bursts of interest have revived their proposal that "Bayesian" methods of probabilistic reasoning should be used in the treatment of evidence at trial.\textsuperscript{36} Other legal academics have resisted, however. Laurence Tribe, for example, has expressed skepticism against applying probabilistic reasoning to jury trials at all.\textsuperscript{37} In any event, Bayes' Theorem seems unlikely to achieve the sort of influence that accompanies law and economics—if indeed it ever achieves any influence at all.

C. Game Theory and the Status of International Law as "Law"

Compared to the use of economics and statistics, a relatively recent attempt to quantify law is the application of game theory to the modeling of international law. Game theory is the mathematical study of strategic decision-making among rational, self-interested beings who compete against or cooperate with one another for gain.\textsuperscript{38} Game theory was originally conceived by the illustrious physicist-mathematician-computer scientist John von Neumann,\textsuperscript{39} but it truly took root as a central concept in economics, where it fit together seamlessly alongside the study of market competition and profit maximization.\textsuperscript{40}

Perhaps the most famous representation of game theory—famous

\textsuperscript{34} Evidentiary applications of Bayes' Theorem are controversial because Bayes' Theorem accounts for the "direction" that each item of evidence is pointing to (for example, guilt or innocence) but not for the weight of each item of evidence. In contrast, any other statistical method that uses sampling takes into account both of these considerations. For a more detailed explanation, see Lea Brilmayer & Lewis Kornhauser, Review: Quantitative Methods and Legal Decisions, 46 U. Chi. L. Rev. 116, 144 (1978).

\textsuperscript{35} See, e.g., Cheryl I. Harris & Kimberly West-Faulcon, Reading Ricci: Whitening Discrimination, Racing Test Fairness, 58 UCLA L. Rev. 73, 119 (2010); Jennifer L. Peresie, Toward a Coherent Test for Disparate Impact Discrimination, 84 Ind. L. J. 773, 776 (2009).


\textsuperscript{37} Laurence H. Tribe, Trial by Mathematics: Precision and Ritual in the Legal Process, 84 Harv. L. Rev. 1329, 1330–31 (1971); see also Brilmayer & Kornhauser, supra note 34.


\textsuperscript{39} John von Neumann, Zur Theorie der Gesellschaftsspiele, 100 Mathematische Annalen 295 (1928).

\textsuperscript{40} See, e.g., Ian Ayres, Playing Games with the Law, 42 Stan. L. Rev. 1291, 1297 (1990).
enough to feature in an Oscar-winning movie— is the Nash Equilibrium, a variant of which is a model nicknamed the "Prisoner's Dilemma." Often further shortened into simply "PD," Prisoner's Dilemma uses a two-by-two matrix to depict two prisoners under interrogation deciding whether to snitch on the other for a crime. In PD, each prisoner ends up ratting the other out for fear that the other will rat them out. Due to these rational decisions, both prisoners get longer sentences than they would have if they had both refused to talk.

<table>
<thead>
<tr>
<th>A Typical Representation of the Prisoner's Dilemma (number of years spent in prison)</th>
<th>Prisoner 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Don't Snitch</td>
<td>Snitch</td>
</tr>
<tr>
<td>Don't Snitch</td>
<td>(0.5, 0.5)</td>
</tr>
<tr>
<td>Snitch</td>
<td>(10, 0)</td>
</tr>
<tr>
<td>(0, 10)</td>
<td>(5, 5)</td>
</tr>
</tbody>
</table>

Figure 1: Payoff Matrix in Prisoner's Dilemma

This two-by-two game theoretic matrix representation has spread well beyond economics and has become a tool for the study of strategic decision-making all across the social sciences.43

It is easy to see why PD is so popular: for starters, it sports an unquestioned pedigree, owing to the originator of game theory (von Neumann) and at least two other Nobel Prize-winning economists.44 The media attention, moreover, has not been limited to Oscar winning movies. For example, an accessibly written, best-selling book (at least by academic standards) employed the intuitively appealing device of a tournament of computer programs competing for the title of most successful decision-making rule, appearing to give empirical foundation to the entire enterprise of applying PD to the study of strategy.45 PD is also attractive for other reasons; for example, it serves as a shorthand description of frequently recurring strategic situations, capturing the conceptual

42. See Daniel Donovan & John Rhodes, The Prisoner's Dilemma Becomes the Lawyer's Dilemma: To Be A Zealous Advocate or A Judas Goat?, 35-JAN. MONT. L. W 8, 10 (2010).
43. See Robert Axelrod, THE COMPLEXITY OF COOPERATION: AGENT-BASED MODELS OF COMPETITION AND COLLABORATION, at xi (1997) ("The two-person iterated Prisoner's Dilemma is the E. Coli of the social sciences. . . ").
essentials without getting lost in the more complex details. Moreover, the paradoxical conclusion supported by PD is fascinating: two rational agents, acting to maximize their self-interest, actually undercut their own interests in ways that even their own awareness of the logical consequences of their decisions cannot prevent.

Unsurprisingly, the PD model has taken a firm grip on the study of international law, where bilateral decision-making scenarios abound—diplomatic negotiations, compliance with customary international law, and wars. In this domain, these two-by-two matrices are used as much more than a descriptive shorthand for international law phenomena. They are employed as a central part of the analysis of one of the most contested questions in international jurisprudence. That question is whether customary international law has a legitimate claim to the status of “law,” which (according to one school of thought) consists of compulsory rules created by a centralized legislator and enforced by a centralized executive. The “neorealism” school of thought contends that, under this definition, international law is not a valid system of laws because conformity to it is better explained by self-interest rather than institutional compulsion:

The first [reason that states abide by Customary International Law, hereinafter CIL] is coincidence of interest . . . regardless of the action taken by others. The second is coercion, where a powerful state . . . forces or threatens to force other states to engage in acts that they would not do in the absence of such force. The third is true cooperation. These cases are best modeled as a bilateral, iterated prisoner’s dilemma in which two states receive relatively high payoffs over the long term . . . . The fourth situation arises when states face and solve bilateral coordination problems. In these cases, if states coordinate on identical or symmetrical actions, they receive higher payoffs than if they do not coordinate.

<table>
<thead>
<tr>
<th>“Coincidence of Interest” (two states are contemplating attacking each other, but their interest lies in not attacking)</th>
<th>State i</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Attack</td>
</tr>
<tr>
<td>State j</td>
<td></td>
</tr>
<tr>
<td>Attack</td>
<td>-2, -2</td>
</tr>
<tr>
<td>Ignore</td>
<td>2, -1</td>
</tr>
</tbody>
</table>

Figure 2: A Two-by-Two Matrix Representing Probable Cooperation Between States

PD provides a different (and allegedly better) explanation than respect
for law does for the apparent conformity of state conduct to international rules of behavior. According to neorealists, as shown in the passage quoted above, states abide by international law because of self-interest, not because the label “international law” makes it “law.” In making this claim, the neorealists have changed the two-by-two Prisoner’s Dilemma matrix from a helpful visual aid to a means of conceptualizing observed international behavior, and from there to a politically charged argument supporting their widely challenged realist agenda.

Note that the transformation of a descriptive tool into a theoretical model propping up a political agenda can be found not only in the application of game theory to law, but also in the application to law of economics. Law and economics scholars have claimed that the economic analysis of law is more scientific than “practical legal reasoning” and that the economic analysis of law is more likely to be correct than legal philosophy because economic analysis “provides more determinate explanations.” This supposed superiority is assumed to entitle economic analysis to impose its premises (e.g., of purely self-interested rationality) upon the law.

In all of these cases, what need be no more than a tool to aid the description of different concepts has become a scientific model that returns “correct” answers that can (and allegedly should) be used to override conventional solutions. This commandeering of simple conceptual aids like the two-by-two matrix in PD, followed by the transformation of these conceptual aids into rigid and politically charged prescriptions, exacts concrete and substantial costs upon both the academic and public discourse, which are documented in the following Parts.

III. MATHEMATICAL MODELING IN LAW: MOTIVATIONS AND CHARACTERISTICS

Mathematical models sometimes seem attractive simply because they sport a veneer of modernity. All things being equal, quantitative methods place a sort of “thumb on the scale,” which adds weight that an otherwise identically persuasive non-quantitative method would not. Is this thumb on the scale warranted? If so, why?

The view of many in academia seems to be that a thumb on the scale is warranted. Legal theorists and social science academics have argued that quantitative methods are more objective and scientific than non-quantitative methods. To the contrary, however, such “quantita-
tive modeling” may merely make a claim seem more science-y. There is no need for putting a thumb on the scales simply because a model is expressed quantitatively. That is the legal equivalent of putting a white lab coat on an attorney. To be clear, mathematical methods can be adopted if they have instrumental value and they would be used correctly. All too often, however, quantification has no attraction other than the trappings of modernity, valued primarily for its ability to lure in unsuspecting onlookers with equations and formulae.

A. Motivations for Quantification: The Beauty of Mathematical Precision

What accounts for many legal academics’ tendency to inappropriately co-opt quantitative models? The costs are clear enough: intellectual rigor in the short run, and possibly the credibility of the legal discipline as a whole, in the long run. These costs are more than enough to demand that the benefits of using quantitative modeling in legal reasoning ought to be tangible, substantial, and weighty.

There can be clear advantages under the right circumstances to working with numbers rather than words.\textsuperscript{52} For example, when the problem presented is legitimately of a numerical nature, it is much more precise and convenient to solve the problem with numbers and mathematical symbols instead of verbal language, even for the most elementary of numerical problems. Figure 3 shows the quadratic formula, which is the generalized statement for the roots of quadratic equations\textsuperscript{53}:

\[
\text{Where } ax^2 + bx + c = 0 (a \neq 0),
\]

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

Figure 3: The Quadratic Formula

Stating the same formula in English words would not only be clumsy and wasteful, but also vulnerable to miscommunication. Mathematics, which has been described as the “only universal language,”\textsuperscript{54} promises much higher precision in part because there is a shared understanding around the globe of what these and other mathematical notations mean.

\textsuperscript{52} Both of your authors are grounded, by reason of personal history, in quantitative disciplines.

\textsuperscript{53} See RON LARSON, ELEMENTARY AND INTERMEDIATE ALGEBRA 635 (2009).

\textsuperscript{54} JEAN-PIERRE CHANGEUX & ALAIN CONNES, CONVERSATIONS ON MIND, MATTER, AND MATHEMATICS 10 (1995).
This precision may be part of what makes quantification attractive to many academics. Solving a problem mathematically is beautiful—mathematical problems (often) have a specific and definite solution, and that solution can be proven scientifically. Thus, when academics quantify a problem that has not been quantified before, they create for themselves an opportunity to solve that problem with mathematically precise and irrefutably scientific answers. Might not many academics secretly crave the ability to say that they are right because science says so?

In the right circumstances, legal academics can indeed articulate legitimate claims of near-mathematical precision. As the discussion below unfolds, more will be said about what these requirements mean. The basics are that the problems they solve must be of a genuinely numerical nature, the quantitative methods used to solve them must be appropriate for the job, and those methods should be used correctly. These criteria are often not satisfied, however; and, when quantitative modeling is not suited for providing a scientifically sound answer to a problem, purporting to solve that problem quantitatively becomes a specious exercise, an opportunity for a dilettante or novice to create a false impression of expertise.

B. Characteristics of Misleading Quantification: Distortion andCircularity

Misleading quantification is often more than just a comical logical fallacy or a benign methodological mistake. The claim that a quantitative model is more useful than a non-quantitative model because the former makes some argument that the latter does not (without verifying the truth of that argument) betrays a desire for expediency, not for intellectual accuracy.

1. Distortion

However tempting the prospect might be, one cannot reduce all problems of normative values, personal preferences, and political ideologies to a matter of calculation in the same way that one can reduce the solution to a quadratic equation into a formula. Claiming to solve a normative legal problem by reference to a quantitative model distorts the true nature of such problems by oversimplifying them. The distortion of a problem leads to specious solutions, which can influence public decision-making when incorporated into scholarship or public policy by people who are likely to be perceived as experts. Calling one’s argument a “model” to divert scrutiny from misleading applications of quantitative reasoning only cloaks the emperor with even more new clothes. One economist has stated that “[a]s economic theorists, we organize our
thoughts using what we call models. The word ‘model’ sounds more scientific than ‘fable’ or ‘fairy tale,’ although I do not see much difference between them.”\(^{55}\) If that is true in economics, how much more true it is in law!

Of course, this “distortion by quantification” does not always use quantitative models that are so brazenly incorrect as to merit the name “fairy tale.” Sometimes, the person attempting to quantify a problem chooses a mathematically sound model but applies it incorrectly. However, the fact that a mathematically sound model was chosen may make the misleading quantification more damaging because the choice of a correct model can disguise the errors in its application. A case in point is Collins, where the prosecution expert chose a model (Bayes’ Theorem) that might have been sound in certain circumstances from a purely theoretical viewpoint but applied it in a disastrously erroneous manner.\(^{56}\) Such incorrect applications of a correct theory can be difficult for laypeople to police; the layperson might be able to google “Bayes’ Theorem” and confirm that it is a reputable model but might not be able to check whether the prosecution “expert” is using the model correctly.

This problem is likely to intensify as the quantitative theories in question become increasingly complex. Opportunities for abuse similar to that committed by the Collins prosecution abound, given that many mathematical methodologies require specialized training to understand correctly.\(^{57}\) As has been said, “sufficiently advanced technology is indistinguishable from magic”;\(^{58}\) the untrained, ordinary person can rarely verify independently the scientific accuracy of highly complicated quantitative methods. This means that the general public must frequently rely on experts, but experts (in this adversary system of ours) are hired by the parties, who often attach a lower priority to scientific integrity than to winning lawsuits.\(^{59}\) Under this system, failing to call out “distortion by quantification” as the pseudoscientific exercise that it is invites litigants to hire faux experts who would overstate, invent, or ignore scientific truth.

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56. See supra Part II, Section B.
57. For example, the Navier-Stokes equations approximate the motion of viscous fluids and are used in weather forecasting. See Peter Constantin & Ciprian Foias, *Navier-Stokes Equations* (1988).
2. Circularity

A second problem that arises with the models described above is circularity. Legal academics' defenses of quantitative legal reasoning sometimes boil down to nothing more than saying "quantitative reasoning is more scientific and precise than verbal reasoning because quantitative reasoning is more scientific and precise than verbal reasoning, according to the same scientific and precise quantitative reasoning." For example, law and economics scholars have claimed that the economic analysis of law "bring[s] to the study of law the virtues of the scientific method of inquiry"; 60 "the methodology of law and economics is essentially the methodology of . . . the scientific method"; 61 and "a reason for the decline of faith in law as an autonomous discipline is the continuing rise in the prestige and authority of scientific and other exact modes of inquiry," such as economics. 62 Claims that law and economics is "scientific"—which sometimes seems to mean nothing more than that its reasoning can be expressed quantitatively—rarely receive much scrutiny.

The scientific method that legal reasoning supposedly employs in law and economics requires mainly that a theory be empirically confirmed. Confirmation requires investigating whether its predictions are borne out—if they are contradicted by experience, then the theory must be rejected. Thus, for example, Richard Posner writes that a "test of a scientific theory is its predictive power," and law and economics predicts that every person is a "rational maximizer of [that person's] self-interest." 63

Instead of putting this predictive hypothesis to test, however, law and economics scholars tend to define rationality so that the prediction never fails: "Rationality is the ability and inclination to choose whatever ends the chooser happens to have selected," 64 and the idea of "well-being incorporates in a positive way everything that an individual might value" and "not limited to wealth or other tangible elements." 65 In other words, the law and economics prediction that everyone is a rational maximizer of self-interest will always be true because whatever is chosen is assumed to maximize the chooser's self-interest. This is not proof that quantitative legal reasoning is scientific or superior to verbal rea-

64. Id at 15.
soning; it is merely a logical fallacy wrapped in *science*-y jargon. Unfortunately, quantification driven by expediency remains strong, perhaps no more so than in the ongoing debate over whether international law should be given the status of "law."

IV. THE DEBATE OVER THE STATUS OF INTERNATIONAL LAW: A SHOWCASE OF MISLEADING QUANTIFICATION AND LOGICAL FALLACIES

The old debate over whether international norms should be treated as "law" has taken on a new form with the advent of quantitative reasoning. The new version of the argument goes roughly as follows: quantitative modeling demonstrates that international law is not "law," but merely a set of cooperative arrangements that reflect the pre-existing common interests of most states. Because it is these common interests that best explain state behavior, international norms cannot be law, which is a set of compulsory rules created by a centralized legislator and enforced by a centralized executive:

The first [reason that states abide by Customary International Law, CIL] is coincidence of interest . . . regardless of the action taken by others . . . . The third is true cooperation. These cases are best modeled as a bilateral, iterated prisoner's dilemma . . . . The fourth situation arises when states face and solve bilateral coordination problems. In these cases, if states coordinate on identical or symmetrical actions, they receive higher payoffs than if they do not coordinate . . . . [O]ur theory is skeptical of the existence of . . . behavioral regularities that are typically thought to constitute CIL.

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<tr>
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<td>Ignore</td>
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</tbody>
</table>

Figure 4: Coincidence of Interest as Depicted by a 2x2 Game Theory Matrix

66. *E.g.*, HANS KELSEN, PRINCIPLES OF INTERNATIONAL LAW 110 (1952) (“International law . . . is to be considered as a valid order only if it is by and large effective. If the norms of international law had no efficacy . . . it would not be possible to conceive of international law as a valid order.”).
68. Id. at 1114-15, 1139.
69. Id. at 1122.
The argument, in short, is that customary international law does not exist because what is often thought to be CIL is a mere coincidence of interests among states. This mere coincidence of interests supposedly better explains the appearance of cooperation than the characterization as "law" does.

The new tool that has been brought to bear is game theory, including the influential model of Prisoner’s Dilemma. Using PD, these claims are “modeled as a bilateral, iterated prisoner’s dilemma” and “test[ed]...[in] three traditional areas of CIL: neutrality, diplomatic immunity, and maritime jurisdiction.”

Explaining the phenomenon coldly and rationally, in terms of states pursuing their own self-interest, is said to reveal that the conventional understanding of international norms as “law” is entirely superfluous.

Legal neorealism thus dons the figurative white lab coat of mathematical modeling by portraying the argument as the result of putting an input (customary international law) into the quantitative model of the Prisoner’s Dilemma. Curiously, however, another school of thought (liberalism) claims to have used the same input and model only to obtain the opposite output: CIL does have a normative content and therefore a legitimate claim to the status of law.

The use of game theory as an underlying methodology for understanding international law presents unique issues regarding the degree to which a descriptive methodology can yield normative conclusions regarding international law... the new realism about international law suffers from a profound misunderstanding about the significance of game theory. In short, the new realism misuses the methodology by concluding that self-interested behavior and normativity are mutually exclusive... the conclusion that the new realists draw from the Prisoner’s Dilemma... is false.

How can it be that neorealists and liberals used the same methodology that they agree is sound, used the same inputs (international law), but ended up with opposite outputs? Did one of the two camps misunderstand...
stand the methodology, as both neorealist and liberal scholars have claimed? Or possibly, did both camps misunderstand the methodology?

We believe that the fault lies with both camps, not for misunderstanding the Prisoner’s Dilemma model, but for committing misleading quantification. In an effort to put a scientific veneer over their ideologically-based opinions on the status of international law, both neorealists and liberals have tried to reduce a debate over value differences to a quantitative problem that can be definitively answered with two-by-two matrices containing eight numbers. Indeed, even though neorealists and liberals are touting opposite conclusions, the reasoning that led to those conclusions begins from the same logical fallacy: the unfalsifiable conclusion.

A. Trivial Truths and Unfalsifiable Conclusions

Both the neorealist and opposing liberal arguments disguise the specious use of quantitative models as a scientifically valid exercise by making their conclusions unfalsifiable. This tactic takes advantage of a concept in mathematical logic known as “trivial truth.”

1. The Fallacy of Trivial Truth

Imagine someone making the argument, “if the Moon is made of cheese, then the Earth revolves around the Sun.” As a matter of common sense, many of us would consider this claim to be false because its premise is false: as far as we are aware, the Moon is not made of curdled milk. In technical terms of mathematical logic, however, this seemingly preposterous argument is true because its conclusion is true: the Earth does revolve around the Sun, regardless of whether the Moon is made of cheese or not. Figure 5, below, illustrates this point:

*If the Moon is made of cheese → the Earth revolves around the Sun.*

<table>
<thead>
<tr>
<th>False premise</th>
<th>True conclusion</th>
</tr>
</thead>
</table>

**True proposition**

Figure 5: An Example of a Trivial Truth in Mathematical Logic

For the same reason, the following statement is true (although trivially so): “If there were dragons on Earth, there are no dragons alive to-

75. See id.; Goldsmith & Posner, Understanding the Resemblance Between Modern and Traditional Customary International Law, supra note 71, at 641.
76. See IMRE LAKATOS, MATHEMATICS, SCIENCE, AND EPISTEMOLOGY: VOLUME 2, PHILOSOPHICAL PAPERS (Gregory Currie & John Worrall eds., 1980).
day.” In other words, a theory of dragons that deduced that no dragons exist today would be true — although trivially so — even if it relied on the preposterous premise that there were dragons at some point in the history of this planet.

The debate over law and economics illustrates the confusion that results from a failure to recognize that technical “truth” of this sort is not a sound basis for construction of a scientific theory. The famed monetarist Milton Friedman committed this exact fallacy in defense of the law and economics movement, which was just getting underway as he wrote. Reasoning in economics was widely criticized as relying on false premises, such as the assumption that individuals maximize their own gains in a “rational” way. Friedman’s defense of this reasoning evaded the dispute over whether people really are rational maximizers of their own gains. Instead of addressing that debate on the merits, Friedman claimed that the truth of that assumption was irrelevant.

It is irrelevant, Friedman claimed, because what matters in judging the value of a theory is whether that theory correctly predicts real-life phenomena, not whether that theory is based on accurate premises. Therefore, the relevant criterion is the truth of the conclusion and not the truth or falsity of the initial premises:

> [The] task [of positive economics] is to provide a system of generalizations that can be used to make correct predictions about the consequences of any change in circumstances. Its performance is to be judged by the precision, scope, and conformity with experience of the predictions it yields . . . .

Viewed as a body of substantive hypotheses, theory is to be judged by its predictive power for the class of phenomena which it is intended to explain. Only factual evidence can show whether it is “right” or “wrong” or, better, tentatively “accepted” as valid or “rejected.” . . . The only relevant test of the validity of a hypothesis is comparison of its predictions with experience. The hypothesis is rejected if its predictions are contradicted . . . it is accepted if its predictions are not contradicted; great confidence is attached to it if it has survived many opportunities for contradiction. Factual evidence can never “prove” a hypothesis; it can only fail to disprove it . . . .

What Friedman fails to recognize here is that a theory whose claim to scientific validity depends exclusively on the technical truth of “predic-

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77. See, e.g., Gary S. Becker, *Irrational Behavior and Economic Theory*, 70 J. POL. ECON. 1, 1 (1962) (“Strong and even violent differences developed . . . . Critics claim that households and firms do not maximize, at least not consistently, that preferences are not well ordered, and that the theory is not useful in explaining behavior.”).


79. *Id.*

80. *Id.* at 8–9.
tions” is not scientifically valid. A body of robust predictions—larger in number and more directly targeting the places where the conventional model and the newly conceived model diverge—is required in order to determine whether the conventional model or the new model proposal is more convincing as “science.”

For example, a theory of dragon extinction is not scientifically valid simply because it predicts (correctly) that fire breathing dragons cannot be found living anywhere in the world today. Assume that the “extinction theory” of dragons provides that dragons became extinct in the year 1400 B.C.; that this was the year in which tall green space invaders arrived on Earth, harvested dragon blood for use as fuel in their flying saucer; and that the invaders killed off all the dragons in the process. We would not want to take the extinction theory of dragons as established truth simply because one of its predictions (that there are no dragons alive today) can be empirically verified.

Scientists do not treat a theory as true simply because it has one prediction that turns out to be empirically correct. Rather, a theory is taken as true only so long as none (or almost none) of its predictions turn out to be false. Those persons who accept our illustrative theory of dragons would be led to believe not only that there are no dragons now, but also that there have been no dragons since shortly after the year 1400 B.C.; that a flying saucer arrived that year with green space invaders; that the space invaders bled the dragons until they died, etc. This theory is not scientifically valid—despite the occasional true prediction—because its premises, taken as a whole, are false.

Whether a particular prediction, once fulfilled, counts towards the truth of a theory (and if so, by how much) is not always easy to determine. Among other things, it matters how many other predictions have been fulfilled empirically, how important the confirmed predictions are to the core of the theory, and whether any disconfirming evidence has been identified. No set formula exists for determining the extent to which a theory has been proven, but one thing is clear, at a minimum: finding one prediction to be borne out empirically is not a case that the theory from which the prediction is drawn is true. That would merely be a fallacy of trivial truth.

Unfortunately, Friedman is not alone in his defense of the fallacy of trivial truth. That group includes more contemporary advocates of law and economics, such as Judges Richard Posner and Frank Easterbrook.81 Judge Posner defended the inaccuracy of the rationality assumption underlying law and economics by stating that “Newton’s law

of falling bodies is unrealistic in its assumption that bodies fall in a vacuum, but it is still a useful theory because it predicts with reasonable accuracy the behavior of a wide variety of falling bodies in the world.”

In a similar vein, Judge Easterbrook wondered: “What’s wrong with models that contain ‘unrealistic’ assumptions? The purpose of any model is to strip away complications, to make intractable problems manageable, to make things simple enough that we can see how particular variations matter.”

Friedman’s, Posner’s, and Easterbrook’s defenses of trivially true models fail. No scientist would claim that the accuracy of the underlying premises is irrelevant, because a central motivation underlying science is to investigate why things happen the way they do. We do not give the status of science to theories based on false premises because no such model can explain why a phenomenon happened, even if the phenomenon was predicted accurately. For example, Aristotle believed that heavier objects fall faster than lighter objects do. Even though this prediction is accurate in that a rock dropped from a tower would reach the ground before a feather dropped from the same height at the same time, no scientist would say that Aristotle accurately explains the reason: the feather hit the ground later than the rock because of air resistance, not because heavier objects “contain[] more earth,” as Aristotle believed.

Posner’s appeal to Newton has no more persuasive force than an appeal to Aristotelian physics. Paul Samuelson explained eloquently as to why Friedman’s defense of inaccurate theories is a fallacy:

[The counterfactual content of a theory is its shame and not its glory . . . . No one expects that anything be perfect, much less a simplified theory. All scientists settle for some degree of approximation . . . . However, the whole force of my attack on the F-twist [the Friedmanite apology for theories with inaccurate premises] . . . is that the doughnut of empirical correctness in a theory constitutes its worth, while its hole of untruth constitutes its weakness. I regard it as a monstrous perversion of science to claim that a theory is all the better for its shortcomings . . . a stubborn fact can kill a pretty theory.]

In fact, Samuelson specifically cites Newton’s theory of universal gravitation (which was cited by Judge Posner as supporting the Friedman defense) in a critique of the Friedman defense:

82. POSNER, ECONOMIC ANALYSIS OF LAW, supra note 63, at 18.
84. CARLOS I. CALLE, SUPERSTRINGS AND OTHER THINGS: A GUIDE TO PHYSICS 36 (2d ed., 2009).
86. CALLE, supra note 84.
One would not jettison Newton's theory until a better one was found to replace it, for the very good reason that Newton did describe many facts correctly. Then along came Albert Einstein. His special theory of relativity described well (but did not "explain") a host of facts. For velocities small compared to the speed of light, Newton's theory came close to duplicating Einstein's. But when the factual chips were down, the simpler Newtonian equations had to be replaced by the Einstein-Lorenz equations because the facts [the greater inaccuracy of the assumptions underlying the Newtonian theory compared to the assumptions underlying Einstein's theory] called for this.

As Samuelson demonstrates, a scientific theory must rely on accurate premises that have a valid logical connection to the conclusions it asserts, and this requirement cannot be brushed away.

2. The Fallacy of Trivial Truth in History

Friedman, Posner, and Easterbrook are not the only ones to have resorted to trivial truth in defense of a specious model. Trivial truth has been used for centuries to create the appearance and credibility of science for many, many pseudoscientists. Consider phrenology, which swept the U.S. in the nineteenth century and was embraced as science by many academics, including some at Harvard University. Phrenology held (among other things) that personality traits were localized to discrete sectors of the brain, and that prominent features in the skull indicated an intensification in personality traits housed in the corresponding parts of the brain.

This sort of reasoning allowed the phrenologist to "prove" phrenology by attaching otherwise accurate conclusions to the phrenologist's absurd premises. If a theory's truth or falsity depends simply on the accuracy of its predictions, then all that one needs to do to substantiate a premise is to attach a truthful statement as an implication, whether the premise and the statement have any logical connection to each other or not. Do you have selfish tendencies? If so, then any line of reasoning concluding with the observation that you are selfish would be correct.

88. Id. at 737.
89. DANNY E. BURTON & DAVID A. GRANDY, MAGIC, MYSTERY, AND SCIENCE: THE OCCULT IN WESTERN CIVILIZATION 193 (2003) ("[I]n the nineteenth century, many scientists ... hailed phrenology as a landmark science."); WILLIAM SCOTT & GEORGE COMBE, THE HARMONY OF PHRENOLOGY WITH SCRIPTURE 226 (1837) ("Dr. [Johann] Spurzheim ... commenced a course of lectures on Phrenology in this city, and soon after, another course at Harvard University, Cambridge ... [and] five lectures before the Medical Faculty, on the anatomy of the brain . . . .").
90. LYDIA FOLGER FOWLER, FAMILIAR LESSONS ON PHRENOLOGY 189 (1847).
If the parts of my skull around the ears are large → I have overly selfish tendencies.

Questionable premise → True conclusion

True proposition

Figure 6: Phrenology Validated Using Trivial Truth

LOCATION AND CLASSIFICATION OF THE FACULTIES.

Figure 7: Application of Phrenology to a Drawing of Brain “Sectors”

As one might predict, a “scientific discipline” that claimed to judge people through the shape of their skulls was used to “validate” racism through “science”:

The phrenologists had argued that the different races ... had different head shapes ... The head shape indicated the cerebral organization and, thus, the size and functional capacity of the brain ... The Anglo-Saxons, the phrenologists had argued, had a perfectly shaped head and, therefore, the largest and most complex brain and also therefore the greatest intellectual capability. Germanic peoples ... were next in line ... [A]ll white people had better-shaped heads, larger brains, and greater intellectual ability than people of color, especially black people ... 92
These ideologies did not contain themselves within the ivory tower. "[I]n the late nineteenth century and even into the twentieth, there were white physicians and white physical scientists, individuals with intellectual ability, who...argu[ed]...that a larger skull indicated a larger brain, and therefore, greater intellectual ability."93 During this same time, popular books had been published on how to quickly identify the dangerous and dishonest.94

Figures 8: Predictions Based on Phrenology

93. Id.
94. Louis Allen Vaught, Vaught's Practical Character Reader 11, 38 (1902); see also Stackpool E. Odell, Heads and How to Read Them: A Popular Guide to Phrenology (1921).
3. The Fallacy of Unfalsifiable Conclusions in the Past and Present

The fallacy of trivial truth takes advantage of a true conclusion to impart a veneer of science to a premise that is pseudoscientific and un-related to that true conclusion. However, as unscientific as the fallacy of trivial truth is, it is more defensible than a related fallacy, the fallacy of unfalsifiable conclusions.

As discussed above, mathematical logic labels as technically “true” any proposition with an accurate conclusion, such as “if the Moon is made of cheese, the Earth revolves around the Sun.” Milton Friedman’s claim that a theory should be judged simply by the accuracy of its implications or predictions is therefore mistaken: even a theory that is scientifically false (such as Aristotelian physics) can be the basis for true predictions. However, Friedman’s methodology would at least reject propositions that return false predictions (and the theories that lead to those predictions), such as “if the Moon is made of cheese, the Earth is made of jelly.” In this sense, the Friedmanite claim that the scientific value of a theory depends exclusively on the truth or falsity of the theory’s predictions contains a partial truth.

But now replace the true conclusion with an unfalsifiable one. Examples of unfalsifiable claims include circular claims (e.g., people are rational maximizers of their own interests). If one chooses a conclusion “Y” that cannot be rebutted factually, then any premise leading to the conclusion Y can be presented as not false—this is just as good as “true” to pseudoscientists, who peddle the proposition as “scientific” no matter how preposterous the premise and the conclusions of the proposition are. This is in contrast to the Friedmanite fallacy of trivial truth, which took advantage of false premises but at least restricted its conclusions to those that were true.

The fallacy of unfalsifiable conclusions is important because many of the ostensibly true statements peddled by pseudoscientists are tautologically true—that is, true by definition—and thus immune to any empirical evidence presented against it. For a historical example of an unfalsifiable theory, take the Roman astronomer Claudius Ptolemy’s theory that the Earth is the center of the solar system. Astronomers who came hundreds of years before Ptolemy had believed that the Sun was the center of the solar system,95 and there were data suggesting that this theory was correct. Proving his theory required Ptolemy to show that the existing data were compatible with his Earth-centric model, which he accomplished by positing that his model allowed for what were

called "epicycles," that is, smaller orbits whose trajectories could be adjusted in size, direction, and speed.96 Thus interpreted, Ptolemy's geocentric model could "account for all apparent movement in the heavens."97 However, by the same "logic," he could equally as well have "proposed that the Moon is the center of the [solar] system" and "come up with a Moon-centered model that fits all data."98

As wrongheaded as the fallacy of unfalsifiable conclusions may seem, it is alive and well in contemporary legal academia. Recall, for example, the proposition that people act rationally as to pursue their own self-interest. This statement might seem to be an empirical claim that is theoretically falsifiable. But, on second glance, it is clear that this statement about people being rational actors is unfalsifiable as interpreted and applied. For whatever a person does—even a counterproductive or irrational action—can be redefined tautologically to be in that person's best interest. If each person is assumed to be the best judge of his or her own best interest, then the claim that persons are rational pursuers of their own best interests is true by definition.

An even more recent example of the fallacy of unfalsifiable conclusions is found in international law academia. A subject of perennial debate in this domain is whether international law is or is not law. The conclusion that "international law is / is not law" is unfalsifiable from a scientific perspective because it does not present any assertions that can be tested objectively and factually; whatever the facts of the situation are, the definition of "law" can be manipulated so that one's prior convictions remain undisturbed. Nevertheless, legal academics on both sides of this debate have claimed, implicitly or explicitly, that their arguments are empirically testable99 and have been shown to be true by the Prisoner's Dilemma model.100 Consider the following:

96. Id. at 41.
97. Id. at 107.
98. Id. at 105.
99. See Norman & Trachtman, supra note 73, at 542-43 ("The purpose of game-theoretic models is . . . to generate testable hypotheses that, once tested, . . . tell us something useful about the world . . . . Theory such as the one articulated in this article must be tested and refined based on empirical observation."); see also Goldsmith & Posner, supra note 46, at 1121 ("The success of our argument, then, depends on both its theoretical plausibility (the subject of this Section) and empirical verification (the subject of the next Section.").
100. See Goldsmith & Posner, A Theory of Customary International Law, supra note 46; see also Ohlin, supra note 74.
If PD shows that states comply with international law because of their own self-interest

\[ \implies \]

**International law is not “law”**

Questionable premise \(\implies\) Unfalsifiable conclusion

**Unfalsifiable proposition**

Figure 9: The “Neorealist”\(^{101}\) Model of International Law

*If PD shows that states comply with international law for reasons other than self-interest*

\[ \implies \]

**International law is “law”**.

Questionable premise \(\implies\) Unfalsifiable conclusion

**Unfalsifiable proposition**

Figure 10: The “Liberal”\(^{102}\) Model of International Law

These propositions are both unfalsifiable and immune to evidence presented against them. There is no single, universally accepted definition of “law” in the same way that there is a single, universally accepted rule that makes one plus one equal two instead of three or the color blue. Because the definition of “law” is based in large part on subjective preferences and opinions, no one can definitively show that “international law is (or is not) law,” contrary to what neorealist and liberal scholars claim. What these scholars actually do in the face of challenge is to tell the challengers that their definition of “law” is incorrect.\(^{103}\)

Recall that neorealist scholars present PD matrices purporting to show that states’ compliance with international law is best explained by self-interest,\(^{104}\) and liberal scholars present PD matrices purporting to show that compliance is explained not only by self-interest but things like “trust” and “reputation” among states.\(^{105}\) For example, the following matrix purports to show that cooperation in international law occurs

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102. See Ohlin, *supra* note 74.
103. Compare *id.*, with Goldsmith & Posner, *Further Thoughts on Customary International Law*, supra note 71 (“[T]here are straightforward game theoretical explanations for [the existence of international law] that do not depend on the normative gravitational pull of CIL.”).
105. See Parisi & Ghei, *supra* note 73, at 119.
not because of any "legal obligation" but because the self-interest of states happens to coincide in taking the same action:\textsuperscript{106}

<table>
<thead>
<tr>
<th>Consequences of Harassing or Protecting the Other State’s Diplomats</th>
<th>State i</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Harass</td>
</tr>
<tr>
<td>State j Harass</td>
<td>2, 2</td>
</tr>
<tr>
<td>Protect</td>
<td>0, 10</td>
</tr>
</tbody>
</table>

**Figure 11: A Prisoner’s Dilemma Model of Compliance with International Law**

Although the scholars who present this model claim that it is "a suitable test case for [their] theory of CIL [that compliance with customary international law is explained by self-interest on the part of states],"\textsuperscript{107} this matrix is just as pseudoscientific and just as pliable as Ptolemy’s theory claiming to show that the solar system revolves around the Earth. Just as Ptolemy could change his equations of planetary orbits to make his theory consistent with facts that seemed to contradict it, the eight numbers in the two-by-two PD matrix can be changed to show that any situation in which states might appear to comply with international law can be better explained by self-interest. In other words, altering the payout structure turns the game of PD into a completely different game—for example, the “battle of the sexes” or “chicken.”

In fact, when one contemplates any reasonably complex real-life situation in international relations, one will soon realize that a two-by-two matrix containing eight numbers is nowhere near rigorous or scientific enough to be able to faithfully represent that real-life situation. For example, PD matrices used by both neorealists and liberals alike assume that state actions in international relations fit into a binary definition of “cooperation” and “defection”—in other words, whether a state is “complying with” or “defying” international law.\textsuperscript{108}

In reality, however, definitions of “compliance” and “defiance” can be both exceedingly hard to craft and also intensely controversial. For an example of how unclear the definitions of cooperation and defection can be, take the dispute between the U.S. and Iran over the latter’s compliance with the Nuclear Non-Proliferation Treaty (“NPT”).

\textsuperscript{106} See Goldsmith & Posner, A Theory of Customary International Law, supra note 46, at 1153.

\textsuperscript{107} Id. at 1151.

has consistently maintained that it has a legal right under the NPT to enrich uranium for peaceful purposes—in other words, Iran has argued that its uranium enrichment activity constitutes “cooperating” in terms of abiding by the NPT. The U.S., however, stated in 2014 that Iran violated the NPT, citing uranium enrichment. Because there is no agreement on what constitutes defection and compliance, this disagreement defies the simplistic binary assumptions of the PD model.

Lest one think that this disagreement is the product of Iran’s willful misinterpretation of the NPT, consider the International Atomic Energy Agency’s (“IAEA”) treatment of uranium enrichment by other states. South Korea covertly enriched uranium to “nearly bomb-grade levels” in 2000, but the IAEA maintained that South Korea had not diverted any nuclear materials to a weapons program and did not inform the U.N. Security Council that South Korea had violated the NPT. In contrast, the IAEA reported Iran to the Security Council for breach of the NPT. Given the disparate treatment of the two states for uranium enrichment, this dispute is a genuine example of a disagreement over the definitions of “cooperation” and “defection” in international relations, as PD would put them. Because of the disagreement over these fundamental definitions, PD models used by neorealists and liberals such as the ones cited above cannot describe a significant subset of events in international relations.

PD brings an illusion of objectivity and precision to things that cannot be quantified or predicted reliably. By quantifying the consequences of state actions, PD matrices such as Figure 11 implicitly assume that the consequences of state actions are predictable—states i and j will reap six units of “profit” if they both choose to protect the

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114. Despite a recent agreement between the U.S. and Iranian governments to reduce sanctions for the latter’s nuclear activities, the disagreement over what constitutes compliance with the deal (endorsed by U.N. Security Council Resolution 2231) continues to persist, as shown by the Iranian Revolutionary Guard’s test in March 2016 of missiles capable of delivering nuclear warheads. Thomas Erdbrink, Iran Tests More Missiles in Message to Israel and Biden, N.Y. TIMES (Mar. 9, 2016), http://www.nytimes.com/2016/03/10/world/middleeast/iran-revolutionary-guards-stage-second-day-of-missile-tests.html [http://perma.cc/G2BS-UD7Y].
other state's diplomats. In reality, however, no real foreign policy professional could claim to be able to predict events with such a high degree of accuracy that the consequences of those events can be reduced to numbers.

Defenders of game theory in international law will probably say that the small deviations that result from difficulties in measurement are likely to be harmless, or nearly so. “Perfection is not the standard,” they might say, and in this they have a valid point. Because it is understood that theory only tracks empirical data approximately, criticisms of the sort just described can be easily deflected as simply nitpicking. The unavoidable difficulties in measuring the inputs and outputs are likely to result in slight discrepancies between what the measurements indicate and what the model predicts. Game theory's defenders can truthfully report that they never claimed that these matrices were perfectly precise.115

These observations, while providing an intuitively appealing defense to criticisms based on measurement difficulties, are wide off the mark in the context of these game theoretic matrices. The reason is that even a very small amount of error in the inputs of game theory matrices can change them into models with completely different incentive structures. This is unlike the situation in many disciplines of science, where the magnitude of the error in the input is often commensurate to the magnitude of error in the output.

By way of illustration, consider the following two matrices, which respectively depict the Prisoner's Dilemma (Figure 12) and the game of Chicken (Figure 13). Chicken goes as follows:

<table>
<thead>
<tr>
<th>Figure 12: Prisoner's Dilemma</th>
<th>Player B</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Snitch</td>
</tr>
<tr>
<td>Player A</td>
<td></td>
</tr>
<tr>
<td>Snitch</td>
<td>-8, -8</td>
</tr>
<tr>
<td>Don't snitch</td>
<td>-10, 0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Figure 13: Chicken game</th>
<th>Player B</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Drive</td>
</tr>
<tr>
<td>Player A</td>
<td></td>
</tr>
<tr>
<td>Drive</td>
<td>-8, -8</td>
</tr>
<tr>
<td>Swerve</td>
<td>-1, 1</td>
</tr>
</tbody>
</table>

115. E.g., Goldsmith & Posner, A Theory of Customary International Law, supra note 46, at 1138 (stating that the matrix depicting coordination is “incomplete”).
Two drivers are driving in opposite directions on a narrow road, headed straight towards one another. Each driver realizes that if s/he swerves then his/her car will end up in the ditch at the side of the road. But this is surely better than smashing head on into a car approaching at high speed. Both drivers want very much to cause the other to swerve, because onlookers will look down on the driver who “chickened out” and admire the one who fearlessly maintained his/her course. In the Prisoner’s Dilemma, there is one Nash Equilibrium (the two players both snitch on each other); in the chicken game, there are two Nash Equilibria (Player A swerves and Player B drives, or Player A drives and Player B swerves).116

But Prisoner’s Dilemma and Chicken are separated only by the different numbers in the payoff matrices. The difference between Figures 12 and 13 is in the numbers in the cells where the two players take different actions (one player snitches while the other does not; one player drives while the other swerves). Changing those numbers completely changes the incentive structures in the model—in PD, the players have an incentive to choose the same action but the players in the chicken game have an interest in choosing different actions. Because even such a small change in the numbers in the matrix drastically changes the nature of the model, numerical imprecision in the PD model cannot be excused for the reason that the imprecision is small.

This is important because typically there is no direct objective evidence of the parties’ payoffs and it becomes necessary to infer the value each player attaches to a particular outcome. If Driver B in Chicken attaches an unusually large negative value to being seen as backing down, and Driver A would rather die on the road than drive her brand-new car into a ditch—regardless of what her opponent does—then the logic of Chicken will not apply. Instead, the game of Chicken turns into a more conflictual interchange, one with little or no possibility of cooperation.

To be based on Prisoner’s Dilemma, a game must have payoffs for the players that warrant characterization as PD. But it is not possible to know a player’s true payoffs in most complex international interactions. Most game theoretic analyses of international interactions find it necessary to simplify, which ordinarily creates imprecision. This may be tolerable in many circumstances, but in the present context it is not because the characterization of which game is applicable in a particular situation cannot be made without already knowing the players’ payoffs.

116. These matrices were drawn in reference to TODD SANDLER, GLOBAL COLLECTIVE ACTION 23-28 (2004).
B. Consequences of Misleading Quantification

The preceding sections discussed various pseudoscientific uses of quantitative reasoning. Although pseudoscientific quantitative reasoning in law is perpetuated mainly by legal academics, adverse consequences of misleading quantification reach far beyond the ivory tower.

1. The Rise of Pseudoscience, the Perversion of the Public Discourse, and the Distortion of Policy

As discussed previously, the essence of misleading quantification is figuratively dressing an attorney in a white lab coat—portraying an unqualified person as an expert in the quantitative sciences. Because these unqualified figures often propagate unscientific notions under a false impression of expertise, they pervert the public discourse by contributing to the rise of pseudosciences that are treated by the larger public as if they were scientific.

Of course, from our vantage point in the twenty-first century U.S., pseudoscientific notions such as phrenology undoubtedly seem unsubstantiated, offensive, and unlikely to repeat themselves. However, that is the point: these ludicrous claims survived as serious propositions for such a long time because they had the imprimatur of expertise, and similarly ludicrous claims continue to survive today because of the same reason. In 1998, then-surgeon and medical researcher Andrew Wakefield published a fraudulent paper in the medical journal The Lancet suggesting that the measles, mumps, and rubella vaccine was linked to autism in children.117 Although the paper began to be questioned in 2004 and was eventually retracted,118 it contributed heavily to the antivaccination movement that survives to this day.119 The mischief that this movement has caused and is likely to cause in the future is substantial: certain parents’ refusal to vaccinate their children has contributed to more frequent outbreaks of preventable diseases.120

As history suggests, false claims of expertise have led to the rise of pseudoscience and the perversion of the public discourse. Moreover, the perversion of the public discourse inevitably affects public policy be-

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because no government is immune from public opinion. Leading contenders in the 2016 Republican presidential primaries—two of them physicians—claimed that parents should be allowed to refuse vaccination for their children if they so desire.121 These positions were undoubtedly influenced by the Republican electorate’s position on mandatory vaccination: a poll revealed that only fifty-nine percent of Republican voters thought that parents should be required to get their children vaccinated.122 Indeed, author of the fraudulent *Lancet* papers and opponent of mandatory vaccination Andrew Wakefield123 was invited in 2015 to testify in the Oregon Senate against a mandatory vaccination bill.124

2. The Difficulty of Identifying Genuine Expertise

When the rise of pseudoscience distorts public policy, properly qualified experts can help reverse the process by educating the public about the falsehoods peddled by the unqualified. However, this plan has a formidable obstacle: non-expert members of the public often cannot distinguish genuine experts from the pseudoscientists when the subject of discussion is sufficiently complex and the qualifications held by the pseudoscientists seem sufficiently *science-y*. Recall that, in the *Collins* case, a prosecutor (presumably without formal training in statistics), was able to peddle a disastrously erroneous application of Bayes’ Theorem through the testimony of a teacher of mathematics.125 The attention given to such erroneous uses of numerical models undoubtedly earned the model even more credibility as legal academics began to advocate the use of Bayes’ Theorem to determine guilt in criminal trials.126

If these non-experts successfully portray themselves as experts to juries in this manner, consequences can be dire. In *Collins*, the lower court’s acceptance of the prosecutor’s specious probabilistic reasoning was corrected by the high court. However, many cases will not get the same chance for correction on appeal, because the vast majority of criminal cases in the U.S. are disposed of via plea-bargaining127 and indigent

122. *Id.*
125. *See supra* Part II, Section B.
126. *See supra* Part II, Section B.
127. GWLADYS GILLIÉRON, PUBLIC PROSECUTORS IN THE UNITED STATES AND EUROPE (2014).
defendants can be forced to forgo appeal, even when a case does go to trial. In such cases, the consequence of choosing misleading quantitative models over verbal reasoning is the possibility of innocent people being convicted.

Although the public can be exceptionally susceptible to acts of misleading quantification, politicians at the highest levels of government or academics can fail to distinguish pseudoscience from genuine expertise. A disastrous example of this phenomenon occurred in China in the late 1950s, where the person responsible for misleading quantification had seemingly impeccable qualifications. That person was Dr. Xuesen Qian, an aerospace engineer who received his doctorate from Caltech, taught at Caltech and MIT, and became a leading figure in the Chinese aerospace and defense programs. In 1958, Qian published “scientific calculations” showing that planting crops more densely and applying more fertilizer would increase the grain yield per mu (0.16 acre) of land twentyfold, from 1,000 kilograms to 20,000. Qian’s “proof” was received enthusiastically by the Communist Party elite and then-Chairman Mao Zedong, who had planned to have the Chinese economy surpass the American economy in fifteen years. The Chinese Academy of Sciences apparently did not oppose Qian’s claims as of 1959, when it convened to discuss “what to do with the extra food” that had not yet materialized.

Beginning in 1958, the Party implemented a widespread campaign of close planting: farmers would plant anywhere between twenty and seventy-eight percent more seeds per plot of land than they had in the past. However, the Party elite was apparently not aware that “close planting and heavy application of fertilizers... tend[s] to increase dis-


130. Xuesen Qian, What Will Be the Grain Yield Per Mu of Land?, CHINA YOUTH NEWS [中国青年报], June 16, 1958, at 4.

131. See KENT G. DENG, CHINA'S POLITICAL ECONOMY IN MODERN TIMES: CHANGES AND ECONOMIC CONSEQUENCES, 1800-2000, at 132 (2011) (“The Emperor's New Clothes were well received at the highest level.”).

132. See DALI L. YANG, CALAMITY AND REFORM IN CHINA: STATE, RURAL SOCIETY, AND INSTITUTIONAL CHANGE SINCE THE GREAT LEAP FAMINE 271 n.126 (1998) (“Qian... wrote that... the output... could reach 20,000 kilograms... Mao Zedong was impressed by Qian's logic.”); RONALD COASE & NING WANG, HOW CHINA BECAME CAPITALIST 15 (2012) (“Qian's article was received by Mao as a theoretical proof of the viability of the Great Leap Forward in agriculture.”).


ease incidence” in rice crops, and “few clear-thinking people dared to point out that deep-plowing and close-planting schemes were at best a waste of energy and at worst a destruction of fertile land.” A blind acceptance of Qian’s pseudoscientific claims, coupled with other missteps, resulted in a “severe nationwide famine between 1959 and 1961, claiming an estimated twenty-seven million lives.”

As these examples show, misleading quantification can be hard to identify for members of the public, government, and even academia. The difficulty of distinguishing pseudoscience from genuine scientific expertise can have consequences that reach far beyond the ivory tower.

V. CONCLUSION – MODERNIZATION: A THUMB ON THE SCALE?

This Article has examined the increasingly popular trend of quantification in law, its signature characteristics, and the dire consequences it can cause. As shown, legal academics have frequently used quantification in cases where it adds no additional insight or persuasive power to the arguments being made, and in cases where quantification disguises judgments based on personal values as mathematical truths derived from scientifically valid procedures. These instances of misleading quantification frequently rely on circular logic to justify themselves, laying claim to the status of science without providing independent proof of scientific status. Yet, as indicated by many legal academics, the fact that an argument is stated in quantitative models and numbers instead of plain English seems to add a thumb on the scale of persuasive power.

One cannot simply proclaim that quantitative reasoning is preferable to verbal reasoning without having some objective basis for determining which result is better. The statement “A and B are different from one another” does not determine that “A is right and B is wrong”; it merely establishes that A and B are different from each other. Arguing that one of the two is more scientific, without independent support for such an assertion, is not only a violation of the most basic principle of scientific conduct. This would also allow the unqualified to use the name of “scientific modeling” for expediency, because anyone could attach the label of science to the methodology that delivers the result that they prefer. In fact, this practice is a pseudoscientific exercise that muddles the public discourse and perverts public policy.

In order to clear ourselves of these transgressions, legal academics should view skeptically all claims—however casual—of quantitative precision and scientific modeling. Fortunately, as the number of legal academics with advanced training in different fields increases, increasing numbers of new academics will have proper training in quantitative methods. The proper policing of quantitative muddling can ultimately contribute to the quality of the discourse in legal academia, by ensuring that quantitative models and methods are used only where appropriate and beneficial.