1997

Antitrust By Chance: A Unified Theory of Horizontal Merger Doctrine

Andrew Chin

Follow this and additional works at: https://digitalcommons.law.yale.edu/ylj

Recommended Citation
Available at: https://digitalcommons.law.yale.edu/ylj/vol106/iss4/4
Antitrust By Chance:
A Unified Theory of Horizontal Merger Doctrine

Andrew Chin

In the three decades since United States v. Von's Grocery Co., in which the Supreme Court enjoined the merger of two grocery chains comprising a total of 7.5% of the Los Angeles market, antitrust law has continued to lack a unified economic framework to guide and justify the structural analysis of horizontal mergers. The development of the modern system of horizontal merger analysis, focusing on market concentration, has not resolved the incoherence and uncertainty in the underlying economic theory. To the contrary, now that structural analysis of market concentration is a requisite element of "virtually every" horizontal merger case, antitrust jurisprudence appears committed to a program of balancing uncertainties, weighing econometric proxies, and applying decision rules to probabilistic statements of fact: a regime of antitrust by chance.

2. Justice Stewart's famous dissenting remark in Von's Grocery, that "[t]he sole consistency that I can find is that in litigation under § 7 of the Clayton Act, the Government always wins," id. at 301 (Stewart, J., dissenting), remains largely unanswered. Although the government today would no longer challenge the proposed merger in Von's Grocery, the courts—and, thus, the merging parties—have continued to follow the guidelines established by the federal antitrust enforcement agencies. See Daniel J. Gifford, The Jurisprudence of Antitrust, 48 SMU L. Rev. 1677, 1701 (1995) (noting that "the [Justice Department's] enforcement criteria have become the effective law"); George A. Hay, Innovations in Antitrust Enforcement, 64 ANTITRUST L.J. 7, 8 n.1 (1995) ("The reason there are so few merger cases actually filed is that when the agencies signal a likely suit, the transaction is often abandoned.").

A horizontal merger is a combination of two or more companies that produce and sell the same product in the same geographic market. See United States v. International Tel & Tel Corp., 306 F. Supp. 766, 774 (D. Conn. 1969). Because horizontal mergers unite direct competitors, they are of particular antitrust concern. See infra notes 27-28 and accompanying text.


4. Jay Greenfield, Beyond Herfindahl: Non-Structural Elements of Merger Analysis, 53 ANTITRUST L.J. 229, 229 (1984) ("Virtually every evaluation of a horizontal merger begins with definition of the relevant product and geographic markets, determination of the market shares of the competitors within those markets and computations of market concentration before and after the acquisition in question ")
In particular, the use of static measures in merger analysis has tended to obscure the fact that antitrust enforcement is directed toward the dynamic behavior of markets. The federal antitrust enforcement agencies have attempted to clarify that the analysis embodied in their Horizontal Merger Guidelines reflects "a dynamic, rather than a static, model of economic performance." They have met with only limited success. As Daniel Oliver, former chairman of the Federal Trade Commission, has observed:

[C]ounsel for merging parties—and even their economic advisers—frequently treat the Guidelines as if they were "a set of rigid mathematical formulas," and a checklist of points to be covered. . . . As a result, doing the arithmetic discussed in the Guidelines, without doing the thinking they require, inevitably tends to conceal the dynamics of any market.

5. The term "Horizontal Merger Guidelines" will be used throughout this Note to refer to the Department of Justice 1992 Horizontal Merger Guidelines, 57 Fed. Reg. 41,552 (1992). See also infra notes 12–13, 54 and accompanying text.


7. Id. A simple example illustrates the potential problem with treating the Horizontal Merger Guidelines as a set of formulas based on a static view of the market. The Horizontal Merger Guidelines use the Herfindahl-Hirschman Index (HHI) to measure market concentration, which is calculated by summing the squares of the market shares held by the respective firms. For example, an industry consisting of two firms with market shares of 70% and 30% has an HHI of 70² + 30², or 5800. See generally Richard A. Miller, The Herfindahl-Hirschman Index as a Market Structure Variable: An Examination for Antitrust Practitioners, 27 ANTITRUST BULL. 593 (1982) (explaining calculation and interpretation of HHI).

Suppose, as in Table 1, an industry consists of four firms, A, B, C, and D, which in Year 0 have market share percentages of 40, 30, 20, and 10, respectively. Consider two scenarios for the market distribution in Year 1: (I) In one scenario, each of the leading firms A and B yields a 5% share to the smaller firms; (II) in another, the distribution is completely inverted, with the large firms utterly unable to maintain their dominance. In assigning a higher concentration index to the market in scenario II than to the market in scenario I, the HHI fails to account for the much greater ease with which firms C and D were able to compete in scenario II. As noted in United States v. Baker Hughes Inc., 908 F.2d 981, 986 (D.C. Cir. 1990), where market shares are "volatile and shifting," concentration statistics evaluated "at any given point in time" may present a misleading picture of a market's competitive structure.

Scenario II is not necessarily unrealistic. As noted in United States v. General Dynamics Corp., 415 U.S. 486, 502 (1974), short-term market share may be unrelated to a company's future ability to compete. In that case, the Supreme Court allowed the acquisition of United Electric by a leading coal producer, because the appropriate measure of future market power was the share of unsold recoverable reserves, not the present market share. United Electric ranked fifth among Illinois coal producers in annual production, but ranked only tenth in reserve holdings. See id.

The bulk of the coal produced is delivered under long-term requirements contracts, and such sales thus do not represent the exercise of competitive power but rather the obligation to fulfill previously negotiated contracts at a previously fixed price. . . . In a market where the availability and price of coal are set by long-term contracts rather than immediate or short-term purchases and sales, reserves rather than past production are the best measure of a company's ability to compete.

Id. at 501–02.

Although these conditions may have been peculiar to the coal industry, courts repeatedly have cited Justice Stewart's opinion in General Dynamics to reduce the defendant's burden in rebutting market share data. See, e.g., Ball Memorial Hosp., Inc. v. Mutual Hosp. Ins., Inc., 784 F.2d 1325, 1336 (7th Cir. 1986) ("Market share is just a way of estimating market power, which is the ultimate consideration. . . . Market share reflects current sales, but today's sales do not always indicate power over sales and price tomorrow."). See generally Baker Hughes Inc., 908 F.2d at 990–91 (describing effect of General Dynamics case on Supreme Court's antitrust jurisprudence).
The purpose of this Note is to demonstrate that the Horizontal Merger Guidelines do express a rational, coherent enforcement policy, informed by the dynamic behavior of market structure. This Note supplies a dynamic market model that enables the reinterpretation of the Horizontal Merger Guidelines as an internally consistent framework of statistical inference. Within this framework, each piece of evidence can be weighed according to its statistical certainty; one factor does not automatically trump another. Specifically, this Note provides and illustrates a method for interpreting the evidentiary factors that have been most frequently dispositive in recent merger case law: market concentration, ease of entry, and concentration trends.

Horizontal merger analysis in the courts usually begins with the calculation of the concentration within the relevant market before and after the proposed merger. The government may establish a presumption of illegality by showing that the merger would "produce a firm controlling an undue percentage share of the relevant market, and [would] result[] in a significant increase in the concentration of firms in that market." Alternatively, the government may demonstrate that the merger would "significantly increase the concentration of an already highly concentrated market." This presumption may be rebutted by "'[n]onstatistical evidence which casts doubt on the persuasive quality of the statistics to predict future anticompetitive consequences,' such as: 'ease of entry into the market, the trend of the market either toward or away from concentration, and the continuation of active price competition.'"

Since 1982, horizontal merger analysis has relied heavily upon the Herfindahl-Hirschman Index (HHI) of market concentration. The federal antitrust enforcement agencies and attorneys general at the state

<table>
<thead>
<tr>
<th>FIRM</th>
<th>YEAR 0</th>
<th>YEAR 1</th>
<th>SCENARIO I</th>
<th>SCENARIO II</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>40%</td>
<td>35%</td>
<td>10%</td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>30%</td>
<td>25%</td>
<td>20%</td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>20%</td>
<td>25%</td>
<td>30%</td>
<td></td>
</tr>
<tr>
<td>D</td>
<td>10%</td>
<td>15%</td>
<td>40%</td>
<td></td>
</tr>
<tr>
<td>HHI</td>
<td>3000</td>
<td>2700</td>
<td>3000</td>
<td></td>
</tr>
</tbody>
</table>

9. Id. at 1219; see also Baker Hughes Inc., 908 F.2d at 983.
10. University Health, 938 F.2d at 1218 (quoting Kaiser Aluminum & Chem. Corp. v. FTC, 652 F.2d 1324, 1341 (7th Cir. 1981)).
11. See supra note 7.
have adopted the HHI for its convenience and power—a single statistic that can be cited both to describe the structure of an industry and to shift a court's presumption away from allowing horizontal mergers between sufficiently large firms within that industry. Despite some initial skepticism, the courts have also come to prefer the HHI as a measure of market concentration in reviewing challenges to horizontal mergers.

The use of the HHI statistic as a proxy for anticompetitive market structure has left the Horizontal Merger Guidelines vulnerable to attack. Comparing the HHI with the previously favored four- and eight-firm concentration ratios, critics have questioned the HHI's algebraic properties and econometric explanatory power. Although these measurement errors are significant, they need not invalidate the HHI statistic as evidence of market power at trial; after all, every piece of evidence carries an element of uncertainty. The HHI of a market may be used appropriately within a framework of statistical inference which gives commensurate weight to presumptions and countervailing evidence. The dominant interpretation of the Horizontal Merger Guidelines, however, is that the presumption of market power is fixed upon market concentration thresholds that were either arbitrarily chosen or based on static models of market behavior.


18. The k-firm concentration ratio of any market is the combined market share of the k largest firms.
22. Former Assistant Attorney General William Baxter, the chief architect of the 1982 DOJ Guidelines, wrote that the thresholds “have no magical qualities” beyond the fact that “we were born with ten fingers and have gotten used to a base ten system.” William F. Baxter, A Justice Department Perspective, 51 Antitrust L.J. 287, 292 (1982).
way, the Horizontal Merger Guidelines fail to quantify standards for rebuttal. This is bad statistics and bad law.

Ultimately, the use of the HHI as a proxy for the competitive structure of a given market entails particular statistical assumptions about the market's dynamic behavior. This Note will supply one possible set of dynamic assumptions that will provide a unifying framework for interpreting the Horizontal Merger Guidelines. Within this framework, the HHI thresholds are meaningful, not arbitrary, and the quantitative standards for rebuttal are well-defined, yet flexible.

Part I of this Note reviews the role of market concentration in the context of federal antitrust enforcement. Part II surveys recent decisions that have used the HHI in analyzing the effects of horizontal mergers. Part III presents the notion of a stochastic market model as a framework for analyzing merger cases. Part IV demonstrates the framework's applicability to the analytical factors indicated in Part II. Part V uses a horizontal merger case, United States v. Country Lake Foods, Inc., to illustrate the framework's potential contribution to the substantive law. Part VI concludes with a discussion of further consequences of the theory.

I. MARKET CONCENTRATION AND ANTITRUST ENFORCEMENT

A horizontal merger has one certain effect: At the moment of the merger, it increases the concentration of the market. It is less clear that the merger creates market power, harms consumers, or makes an eventual monopoly more likely. Even so, the passage of the Sherman Act established as public policy the nation's concern with the empirical relationship between horizontal mergers and monopoly market power. Consequently, market concentration has always been cognizable under the antitrust laws as an indicator of market power.

Section 7 of the Clayton Act (as amended in 1950) prohibits any merger or acquisition where "the effect of such acquisition may be substantially to lessen competition, or to tend to create a monopoly." The Department of Justice is authorized to prevent violations through equitable proceedings in the federal district courts. The Federal Trade Commission may enforce section 7 through administrative proceedings in the agency and injunctive actions in

24. See Hsu, supra note 3, at 80.
25. The proposed framework incorporates the stochastic market model developed by Robert Gibrat. See infra Part III.
27. See 4 PHILLIP AREEDA & DONALD F. TURNER, ANTITRUST LAW § 901, at 2 (1980) ("[I]t was a series of mergers, virtually monopolizing several leading industries, that was primarily responsible for the passage of the Sherman Act.").
29. See id. § 25.
the courts. Private parties may also seek injunctive relief and treble damages from section 7 violations in the federal courts.

From the earliest years of section 7 jurisprudence, antitrust policy has been directed toward mergers between sufficiently large firms in highly concentrated markets. *Brown Shoe Co. v. United States* interpreted section 7 as a barrier against “the rising tide of economic concentration” in American industry that applied even “when the trend to a lessening of competition in a line of commerce was still in its incipiency.” Subsequent Supreme Court decisions elaborated a burden-shifting, structural analysis of horizontal mergers. In *United States v. Philadelphia National Bank*, the Court held that a rule of presumptive illegality applies to a horizontal merger that causes a “significant increase in [market] concentration” and produces a firm with an “undue percentage [market] share.” Extending this rule in *United States v. Aluminum Co.*, the Court held that horizontal mergers or acquisitions causing even slight increases in concentration are presumptively illegal in highly concentrated markets. In *United States v. Von’s Grocery Co.*, mergers causing slight increases in concentration were also ruled illegal in markets where there is a significant “trend toward concentration.”

Market concentration has retained its central importance as a structural measure of competition. In other respects, however, antitrust enforcement regarding horizontal mergers has evolved substantially in the years since *Von’s Grocery*. First, the Department of Justice and the Federal Trade Commission, in assuming primary responsibility for screening all mergers between large firms for anticompetitive effect, have adopted guidelines establishing market concentration thresholds below which mergers will not be challenged. Second, merger analysis now includes a broad range of market factors, including ease of entry, that may be relevant to a finding of anticompetitive effect.

The Hart-Scott-Rodino Antitrust Improvements Act of 1976 requires

---

30. See id. §§ 21, 53(a).
31. See id. § 26.
33. Id. at 317. The Court explained: “Congress saw the process of concentration in American business as a dynamic force; it sought to assure the Federal Trade Commission and the courts the power to brake this force at its outset and before it gathered momentum.” Id. at 317–18.
35. Id. at 363.
37. See id. at 279.
39. Id. at 277.
40. The Horizontal Merger Guidelines continue to confer a presumption of anticompetitive effect on a merger that increases concentration in a concentrated market. See Department of Justice 1992 Horizontal Merger Guidelines, 57 Fed. Reg. 41,552 (1992); *infra* text accompanying notes 55–58.
that the parties to a merger between sufficiently large firms\textsuperscript{43} file "Pre-Merger Notification" forms with the Federal Trade Commission and the Department of Justice. Recognizing their limited resources, these agencies adopted a simplified two-stage approach to merger review that was formalized in the agency guidelines. The first stage identified markets in which concentration was initially high and would increase significantly after a proposed merger. The second stage determined whether firms could actually behave collusively in those markets, taking into account a variety of market factors, including ease of entry. Not surprisingly, the case law has evolved in response to the agency guidelines. Today, "virtually every" horizontal merger case begins with a computation of pre- and post-merger market concentration, before taking into account other structural and nonstructural factors.\textsuperscript{44}

Of these rebuttal factors, ease of entry has received the most extensive attention from the courts.\textsuperscript{45} To date, however, the courts have avoided taking an analytical approach that quantitatively balances each factor in the Horizontal Merger Guidelines against the weight of the market concentration statistics, preferring simply to let ease of entry "trump" concentration.\textsuperscript{46} For instance, in United States v. Waste Management, Inc.,\textsuperscript{47} the Second Circuit found that entry into the trash collection business was sufficiently easy to overcome the presumed anticompetitive effect evidenced by market concentration statistics.\textsuperscript{48} In United States v. Calmar Inc.,\textsuperscript{49} a New Jersey district court reached a similar decision, finding that any firm in the injection molding business could enter the pump sprayer market.\textsuperscript{50} These rulings were underscored in United States v. Baker Hughes Inc.,\textsuperscript{51} a D.C. Circuit decision written by then-Judge Clarence Thomas for a panel including then-Judge Ruth Bader Ginsburg, which found easy entry in a highly concentrated market sufficient to overcome post-merger market power\textsuperscript{52} despite a lack of evidence that entry would be "quick and effective."\textsuperscript{53}

Thus, the recent case law on horizontal mergers might be expected to conform closely to the Horizontal Merger Guidelines' market concentration thresholds, except where the HHI statistics have been trumped by other factors. The survey of recent horizontal merger cases in the next Part confirms this

\textsuperscript{43} Generally, the Act applies if either firm is engaged in interstate commerce, see id. § 18a(a)(1), and a party with annual net sales or total assets of at least $100 million is acquiring a party with annual net sales or total assets of at least $10 million, see id. § 18a(a)(2)(A).

\textsuperscript{44} See Greenfield, supra note 4, at 229.

\textsuperscript{45} See Hsu, supra note 3, at 75 ("The most significant new issue in merger analysis is ease of entry.").

\textsuperscript{46} See id.

\textsuperscript{47} 743 F.2d 976 (2d Cir. 1984).

\textsuperscript{48} See id. at 982.

\textsuperscript{49} 612 F. Supp. 1298 (D.N.J. 1985).

\textsuperscript{50} See id. at 1306.

\textsuperscript{51} 908 F.2d 981 (D.C. Cir. 1990).

\textsuperscript{52} See id. at 987.

\textsuperscript{53} Id. at 988.
expectation, and suggests the desirability of an analytical justification for the thresholds and a quantitative framework for balancing market concentration with ease of entry and other evidence.

II. THE HHI IN THE COURTS

The Department of Justice (DOJ), Federal Trade Commission (FTC), and National Association of Attorneys General (NAAG) Horizontal Merger Guidelines all use the HHI to indicate whether a particular horizontal merger will result in a concentrated market and thereby increase the likelihood of a challenge. Since the first appearance of the HHI in each of the guidelines, the substantive rules have remained unchanged. The guidelines uniformly regard a market in which the post-merger HHI is below 1000 as "unconcentrated," between 1000 and 1800 as "moderately concentrated," and above 1800 as "highly concentrated." A merger potentially "raise[s] significant competitive concerns" if: (1) it produces an increase in the HHI of more than 100 points in a moderately concentrated market; or (2) it produces an increase in the HHI of more than 50 points in a highly concentrated market. Further, a merger is presumed "likely to create or enhance market power or facilitate its exercise" if it produces an increase in the HHI of more than 100 points in a highly concentrated market.

Recognizing that the post-merger HHI may overstate market power, the Horizontal Merger Guidelines include other factors that may be considered in assessing the potential anticompetitive effects of a merger: (1) coordinated interaction among merging or competing firms; (2) product differentiation by merging firms; (3) capacity constraints on competing firms; (4) ease of entry for new firms; (5) efficiencies claimed by merging firms; and (6) imminent failure of a merging firm. In addition to raising these issues,

---

54. See supra notes 12–13. The National Association of Attorneys General has promulgated its own procedure for merger analysis, but uses the HHI statistic as an indicator of market concentration.

The DOJ and NAAG Horizontal Guidelines adopted nearly identical concentration level standards for mergers subject to section 7 of the Clayton Act. Their main differences concern: 1) policy assumptions, 2) market definition methodologies, and 3) factors other than concentration levels which should affect the decision whether to challenge a merger.


56. See id.

57. See id.

58. Id.

59. See id. at 41,558–59.

60. See id. at 41,559.

61. See id.

62. See id. at 41,561.

63. See id. at 41,562–63.

64. See id.
a defendant may challenge the geographic and product market definitions used to calculate the post-merger HHI.65

Table 2 presents the results of reported horizontal merger cases that have used the HHI in merger analysis. The cases are generally arranged in decreasing order of post-merger HHI. Each listing notes whether the merger was held lawful (L) or unlawful (U) under section 1 of the Sherman Act66 (S), section 7 of the Clayton Act67 (C), or both (C, S). Decisions concerning preliminary injunctions are indicated by the letter "p."

Of the twenty-two merger challenges cited, fourteen were upheld and eight were dismissed. In all of the successful challenges, the increase in HHI and the post-merger HHI exceeded the DOJ thresholds. In all but one of the dismissed challenges, either the DOJ thresholds were not violated (two cases)68 or the market concentration statistics were successfully rebutted by evidence showing ease of entry (four cases)69 or concentration trends (one case).70 With only one exception, FTC v. Freeman Hospital,71 the outcome of each case in Table 2 was consistent with the burden-shifting analytical framework supplied by the Horizontal Merger Guidelines.

The overall impression that emerges from an examination of Table 2 and its accompanying footnotes is that of a case law heavily influenced by the HHI thresholds for presumptive illegality, and therefore heavily reliant on the

---

65. See, e.g., United States v. Country Lake Foods, Inc., 754 F. Supp. 669, 672 (D. Minn. 1990) (rejecting government's geographic market definition and noting ease of entry in broader market), see also infra Part V.


67. Id. § 18.


71. 911 F. Supp. 1213 (W.D. Mo. 1995), aff'd, 91 F.3d 260 (8th Cir. 1995). This preliminary injunction hearing focused on "a battle of the experts" over the relevant geographic market definition. See id. at 1217. In a decision affirmed by the Eighth Circuit, the district court accepted the defendant's proposed service area, which provided a basis for calculating pre- and post-merger HHI statistics See id. at 1221. The post-merger HHIs for beds, admissions, and patient census were 1322, 1496, and 1624, respectively, reflecting increases due to the merger of 189, 251, and 222, respectively See id. at 1222. Thus, even on the defense expert's figures, the merger would result in significant increases in concentration in three concentrated markets, exceeding the DOJ thresholds. Finding that "the post-merger levels of HHI will not exceed 1800," the court nevertheless concluded that "the FTC has failed to demonstrate that competition will be substantially lessened as a result of this consolidation." Id. In determining that the government had failed to prove the merger's prima facie illegality, the district court thus applied a higher standard to the HHI statistics than that provided in the Horizontal Merger Guidelines.

Thus far, Freeman Hospital has been cited only to support the view that "[d]efining geographic markets for medical care has proven no more tractable than geographic markets in general." Methodist Hosps., Inc. v. Sullivan, 91 F.3d 1026, 1029 (7th Cir. 1996). It remains to be seen whether Freeman Hospital will set a precedent for further judicial departure from the analytical structure of the Horizontal Merger Guidelines.
### Table 2. Summary of Horizontal Merger Cases Using the HHI

<table>
<thead>
<tr>
<th>CASE</th>
<th>CLAIM</th>
<th>RESULT</th>
<th>MARKET</th>
<th>PRE-MERGER HHI</th>
<th>POST-MERGER HHI</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alliant Techsystems</td>
<td>C</td>
<td>U, p</td>
<td>120mm tactical ammunition, U.S.</td>
<td>n/a</td>
<td>10000</td>
</tr>
<tr>
<td>Tasty Baking</td>
<td>C, S</td>
<td>U, p</td>
<td>Snack cakes and pies, 2 regions, 4 cities (min) (max)</td>
<td>2354</td>
<td>2797</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>5197</td>
<td>6420</td>
</tr>
<tr>
<td>Ivaco</td>
<td>C</td>
<td>U, p</td>
<td>Automatic tampers for rail tracks, U.S.</td>
<td>3549</td>
<td>5809</td>
</tr>
<tr>
<td>Bon-Ton Stores</td>
<td>C, S</td>
<td>U, p</td>
<td>Traditional department stores, metropolitan area</td>
<td>3395</td>
<td>5074</td>
</tr>
<tr>
<td>Rice Growers</td>
<td>C</td>
<td>U</td>
<td>California paddy rice, Pacific region</td>
<td>3276</td>
<td>4874</td>
</tr>
<tr>
<td>United Tote</td>
<td>C</td>
<td>U</td>
<td>Totalisators, North America</td>
<td>3940</td>
<td>4640</td>
</tr>
<tr>
<td>Rockford Memorial</td>
<td>C, S</td>
<td>U, p</td>
<td>Hospital beds, 3-county area</td>
<td>2555</td>
<td>4603</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Hospital admissions</td>
<td>2789</td>
<td>5111</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Hospital days</td>
<td>3026</td>
<td>5647</td>
</tr>
</tbody>
</table>

72. Table 2 lists many of the horizontal merger decisions that have cited the DOJ Guidelines pertaining to the HHI. Although probably not exhaustive, it is the result of an extensive search and appears to be representative of the relevant case law. As Table 2 serves in part to update the table of horizontal merger decisions provided in Areeda and Turner's 1980 treatise, the list is organized according to similar conventions. See 4 AREEDA & TURNER, supra note 27, ¶ 909b, at 33–51. The main difference is that the individual market share and concentration ratio data have been replaced by the pre- and post-merger HHI indices used in the application of the guidelines.


76. Bon-Ton Stores, Inc. v. May Dep't Stores Co., 881 F. Supp. 860, 875–76 (W.D.N.Y. 1994) (finding that "market concentration would violate the recognized guidelines and would suggest dire anticompetitive consequences").


78. United States v. United Tote, Inc., 768 F. Supp. 1064, 1069 (D. Del. 1991) (finding HHI statistics established section 7 violation where barriers to entry were high and other considerations would not be effective to redress anticompetitive effect).

79. United States v. Rockford Mem'l Corp., 717 F. Supp. 1251, 1280, 1287 (N.D. Ill. 1989) ("Based on an examination of the relevant market's concentration, barriers to entry, nature of competition, and market participants, the court finds that the post-merger market is ripe for anti-competitive behavior."). aff'd, 898 F.2d 1278 (7th Cir. 1990).
<table>
<thead>
<tr>
<th><strong>CASE</strong></th>
<th><strong>CLAIM</strong></th>
<th><strong>RESULT</strong></th>
<th><strong>MARKET</strong></th>
<th><strong>PRE-MERGER HHI</strong></th>
<th><strong>POST-MERGER HHI</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>Baker Hughes</td>
<td>C</td>
<td>L</td>
<td>Hardrock hydraulic underground drilling rigs, U.S.</td>
<td>2878</td>
<td>4303</td>
</tr>
<tr>
<td>University Health</td>
<td>C</td>
<td>U, p</td>
<td>Inpatient services, acute care hospitals, Augusta area</td>
<td>≤2570</td>
<td>3200</td>
</tr>
<tr>
<td>Waste Management</td>
<td>C</td>
<td>L</td>
<td>Waste collection, Dallas area</td>
<td>~3090</td>
<td></td>
</tr>
<tr>
<td>Calmar</td>
<td>C</td>
<td>L, p</td>
<td>Plastic pump sprayers, U.S.</td>
<td>2302</td>
<td>3040</td>
</tr>
<tr>
<td>Country Lake Foods</td>
<td>C</td>
<td>L, p</td>
<td>Fluid milk, Minneapolis area</td>
<td>2186</td>
<td>2832</td>
</tr>
<tr>
<td>Illinois Cereal Mills</td>
<td>C</td>
<td>U, p</td>
<td>Dry corn milled food products, U.S.</td>
<td>2114</td>
<td>2606</td>
</tr>
<tr>
<td>Bass Brothers</td>
<td>C</td>
<td>U, p</td>
<td>Carbon black production, U.S.</td>
<td>1669</td>
<td>2238</td>
</tr>
<tr>
<td>Central State Bank</td>
<td>S</td>
<td>L</td>
<td>Commercial banking, two counties</td>
<td>2161</td>
<td>2182</td>
</tr>
</tbody>
</table>

80. United States v. Baker Hughes Inc., 908 F.2d 981, 987 (D.C. Cir. 1990) (noting that district court concluded that "entry was likely, particularly if [the merger] were to lead to supracompetitive pricing").
82. United States v. Waste Management, Inc., 743 F.2d 976, 983 (2d Cir. 1984) ("[Market entry] is so easy that any anti-competitive impact of the merger before us would be eliminated more quickly by such competition than by litigation."). The HHI figure was estimated from market share data. See id. at 981.
84. United States v. Country Lake Foods, Inc., 754 F. Supp. 669, 673 (D. Minn. 1990) (finding HHI statistics insufficient to establish section 7 violation where several remote competitors were prepared to enter government's geographic market).
<table>
<thead>
<tr>
<th>CASE</th>
<th>CLAIM</th>
<th>RESULT</th>
<th>MARKET</th>
<th>PRE-MERGER HHI</th>
<th>POST-MERGER HHI</th>
</tr>
</thead>
<tbody>
<tr>
<td>Christian Schmidt&lt;sup&gt;89&lt;/sup&gt;</td>
<td>C</td>
<td>U, p</td>
<td>Beer, Upper Midwest</td>
<td>1746</td>
<td>2120</td>
</tr>
<tr>
<td>American Stores&lt;sup&gt;90&lt;/sup&gt;</td>
<td>C</td>
<td>U, p</td>
<td>Supermarkets, 14 MSAs (average for all MSAs)</td>
<td>1795</td>
<td>2040</td>
</tr>
<tr>
<td>Freeman Hospital&lt;sup&gt;91&lt;/sup&gt;</td>
<td>C, S</td>
<td>U, p</td>
<td>Hospital beds, 13 county area</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Hospital admissions</td>
<td>1133</td>
<td>1322</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Hospital patient census</td>
<td>1245</td>
<td>1496</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1402</td>
<td>1624</td>
</tr>
<tr>
<td>Consolidated Gold Fields&lt;sup&gt;92&lt;/sup&gt;</td>
<td>C, S</td>
<td>U, p</td>
<td>Gold sales, noncommunist countries</td>
<td>735.1</td>
<td>1223.2</td>
</tr>
<tr>
<td>LTV&lt;sup&gt;93&lt;/sup&gt;</td>
<td>C</td>
<td>U</td>
<td>Hot rolled carbon and alloy strip, U.S.</td>
<td>871</td>
<td>1047</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Cold rolled carbon and alloy strip, U.S.</td>
<td>953</td>
<td>1146</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Stainless cold rolled sheet and strip, U.S.</td>
<td>2190</td>
<td>2898</td>
</tr>
<tr>
<td>Rothery Storage&lt;sup&gt;94&lt;/sup&gt;</td>
<td>S</td>
<td>L</td>
<td>Van lines, U.S.</td>
<td>520</td>
<td>≤868</td>
</tr>
</tbody>
</table>

---


91. FTC v. Freeman Hosp., 911 F. Supp. 1213, 1222 (W.D. Mo. 1995), aff’d, 69 F.3d 260 (8th Cir. 1995). Post-merger levels of HHI below 1800 were a sufficient basis for the court to conclude that “the FTC has failed to demonstrate that competition will be substantially lessened as a result of this consolidation.” Id.


94. Rothery Storage & Van Co. v. Atlas Van Lines, Inc., 792 F.2d 210, 220–21 (D.C. Cir. 1986) (pre-merger HHI of 520 was “low on the range of unconcentrated markets,” id. at 220 (emphasis omitted); post-merger HHI of 868 was “still well below the top border for ‘unconcentrated markets,’” id.; and HHI statistics made plaintiff’s characterization of market as tight oligopoly “merely absurd,” id. at 221).
internal consistency and precision of the underlying theory. In addition, the case law pertaining to countervailing factors such as ease of entry and concentration trends appears to rely on a small number of necessarily idiosyncratic decisions. The five decisions surveyed here in which evidence of market concentration was successfully rebutted thus warrant further discussion.

In United States v. Waste Management, the Second Circuit reversed a district court decision barring a merger between two Texas waste disposal firms. Relying on General Dynamics for the proposition that market share data may not reflect market power, the court concluded that market entry "is so easy that any anti-competitive impact of the merger before us would be eliminated more quickly by such competition than by litigation." To emphasize the relevance of ease of entry as an affirmative defense, the court noted its inclusion as a factor in the Horizontal Merger Guidelines: "If the Department of Justice routinely considers ease of entry as relevant to determining the competitive impact of a merger, it may not argue to a court addressing the same issue that ease of entry is irrelevant." The Waste Management decision was followed in United States v. Calmar Inc., where the district court allowed a merger in a highly concentrated market, because ease of entry ensured that "the producers in the market could not long sustain an unjustified price increase." Noting a lack of entry barriers and chronicling the history of entry into the market since the 1950s, the court concluded that despite the merger, "it is highly unlikely that the market dynamics will change. . . . The reason for this is the ease of entry."  

95. Areeda and Turner caution against interpreting their own table as a claim that market share data were dispositive in all of the decisions cited. "Highlighting market shares and concentration ratios, especially where no other factors are mentioned, may invite the inference that the tribunal’s decision depended solely on those numbers. Such an inference would sometimes be overdrawn." Areeda & Turner, supra note 27, ¶ 909b, at 33. Although Table 2 indicates an even stronger relationship between decisions and market concentration statistics, the same caution applies here.


97. 743 F.2d 976 (2d Cir. 1984).
98. 415 U.S. 486 (1974); see supra note 7.
99. 743 F.2d at 983.
100. Id.
102. Id. at 1301.
103. See id. at 1305.
104. See id. at 1306.
105. Id.
In *United States v. Baker Hughes Inc.*, the D.C. Circuit affirmed a district court decision disregarding high market concentration statistics where market shares were "volatile and shifting." The product market at issue, hydraulic drilling rigs, was so "miniscule" in the United States that a single contract "could catapult a firm from last to first place." The *Baker Hughes* court also rejected the "quick and effective" standard suggested by *Waste Management*, noting that "evidence regarding specific competitors and their plans... is rarely available" to defendants. To rebut the government's prima facie case, it was sufficient for the district court to find that future entry was likely.

In *New York v. Kraft General Foods, Inc.*, a merger that barely exceeded the HHI thresholds was subjected to a sophisticated economic analysis. Noting that the Horizontal Merger Guidelines "are helpful in providing an analytical framework for evaluating an acquisition, but... are not binding upon the court," the district court revived 1970s case law to justify its consideration of concentration trends. Noting a "long term trend of reduced concentration in the [ready-to-eat] cereal industry," the court determined that the market share data did not create a presumption of a Clayton Act section 7 violation. Lacking this presumption, the state was unable to provide a persuasive theory of anticompetitive effect.

The remaining case, *United States v. Country Lake Foods Inc.*, is the case study that is discussed in Part VI.

### III. THE GIBRAT MODEL

As the cases reviewed in Part II indicate, merger analysis in modern antitrust jurisprudence relies on a combination of static structural measures (e.g., market concentration and entry barriers) and assumptions about the dynamics of market structure (e.g., that entry, market share volatility, and/or the trend toward deconcentration will continue). For example, when the *Baker*...
Hughes court argued that market shares were so "volatile and shifting"\textsuperscript{118} that the HHI statistic was an unreliable indicator of the market's competitive structure, it made a quantitative assertion about measurement error in a dynamic system. Similarly, the Kraft General Foods decision, in noting a trend toward deconcentration, suggested that the structural dynamics of the cereal market effectively raised the state's burden in showing anticompetitive effect from HHI statistics.\textsuperscript{119}

Such assertions implicitly appeal to dynamic predictive models of market structure in which the parameters governing changes in market share and market concentration are inferred from past history and present conditions. Without an explicit, well-defined, dynamic model of market structure, however, it is impossible to assess the predictive power of these analyses, and therefore imprudent to apply the resulting decisions as precedent.

The remainder of this Note supplies one possible dynamic model of market structure and illustrates how it can be used as the basis for a balancing analysis that reinterprets the Horizontal Merger Guidelines as a unified system of statistical inference. The proposed framework quantifies measurement error in the HHI statistic by modeling market structure as a stochastic process first specified by Robert Gibrat.\textsuperscript{120} In this dynamic, stochastic model, a market consists of $n$ initially equal firms\textsuperscript{121} that are subject to independent, identically distributed, randomly varying annual growth rates. Observed market concentration in the Gibrat model consists of two components: a stochastic component depending solely on the known probability distribution of the growth rates—hereinafter referred to as the competitive structure of the market\textsuperscript{122}—and a purely random component, which depends solely on chance. Within this framework, the task of the courts in horizontal merger cases is to draw valid statistical inferences about the competitive structure of markets from observed levels of market concentration.

The Gibrat framework represents a radical departure from the static, classical theories traditionally cited in support of the HHI,\textsuperscript{123} and it is by no means the only possible set of assumptions governing dynamic market behavior. The analysis of a particular merger case within this framework will be conclusive only to the extent that the underlying theory provides a suitable structural model for the relevant market.

\begin{small}
\begin{enumerate}
\item \textsuperscript{118}908 F.2d 981, 986 (D.C. Cir. 1990).
\item \textsuperscript{119}See 926 F. Supp. at 363.
\item \textsuperscript{120}See Michael Kalecki, \textit{On the Gibrat Distribution}, 13 \textsc{Econometrica} 161 (1945)
\item \textsuperscript{121}As applied to a given market definition, the parameter $n$ may include both existing firms and potential entrants, provided that all firms face the same distribution of growth rates.
\item \textsuperscript{122}Cf. YUI IJIRI & HERBERT A. SIMON, \textsc{Skew Distributions and the Sizes of Business Firms} 150 (1977) ("If firm sizes are determined by a stochastic process, then the appropriate way to think about public policy in this area is to consider the means by which the stochastic process can be altered, and the consequences of employing these means.").
\item \textsuperscript{123}See George J. Stigler, \textit{A Theory of Oligopoly}, 72 \textsc{J. Pol. Econ.} 44, 45-48 (1964) (relating HHI to likelihood of effective collusion).
\end{enumerate}
\end{small}
For the purposes of this Note, however, the Gibrat theory is a structurally and empirically adequate dynamic model of market structure. The characteristic feature of the Gibrat model, the hypothesis that large and small firms face the same distribution of growth rates, is inaccurate for some industries, but appropriate for the domain of antitrust policy. Generally, industries in which large firms systematically face significantly higher growth prospects are natural monopolies requiring regulation to maintain competition, and industries that structurally favor small firms are naturally competitive. The stochastic nature of the Gibrat theory also makes it well-suited to a merger analysis that includes probabilistic statements about firm behavior, including the likelihood of market entry.

In their leading textbook on market structure, Scherer and Ross describe a simulation experiment that periodically measured the four-firm concentration ratio of a market based on the Gibrat model with \( n = 50 \). The authors observed that "[patterns resembling the concentrated structures of typical manufacturing industries emerge within a few decades."

Many other studies have provided a sufficient empirical basis for concluding that "chance plainly does play a role in company growth and . . . actual firm size distributions often correspond to those predicted by stochastic growth models." In contrast, classical deterministic economic theory "provides no explanation for the repeated appearance of [observed market share] distributions," and therefore, "either predicts the facts incorrectly or . . . makes no prediction at all." Given that deterministic economic models of the HHI are often founded on "arbitrary and internally inconsistent" assumptions, a stochastic framework that recognizes the chance element in market development seems comparatively realistic and appropriate.

Scherer and Ross attempt to explain the concentration observed in their simulation experiments by the "run of luck" enjoyed by the leading firms: "[O]nce a firm has, by virtue of early good luck, placed itself among the industry leaders, it can achieve additional market share gains if it should happen again to be luckier than average (as it will be in roughly half of all

---

125. The Department of Justice has faced difficulties in attempting to analyze likelihood of entry within the Merger Guidelines' static, deterministic microeconomic framework. See Hsu, supra note 3, at 83 & n.38.
127. See IJIRI & SIMON, supra note 122, at 110; RICHARD R. NELSON & SIDNEY G. WINTER, AN EVOLUTIONARY THEORY OF ECONOMIC CHANGE chs. 12–14 (1982).
128. SCHERER & ROSS, supra note 126, at 146.
129. IJIRI & SIMON, supra note 122, at 10.
130. Cohen & Sullivan, supra note 17, at 486. One standard assumption is that each firm in a market behaves as though the output of every other firm is constant (the Cournot hypothesis). See id. at 487.
This explanation is obviously incomplete because in the other half of all cases a leading firm will lose market share.

The analysis in the Appendix provides a more persuasive and informative explanation for Scherer and Ross's experimental results by estimating the overall distribution of the concentration of Gibrat markets. In the analysis, I consider a Gibrat market defined with the following properties: (1) the market consists of \( n \) equal-sized firms in Year 0; (2) if \( e_{ij} \) represents the rate of growth in year \( i \) of firm \( j \), \( 1 \leq i \leq n \), then the random variables \( z_j = \log(1 + e_{ij}) \) are normally distributed with mean \( z \) and variance \( \sigma^2 \). For this market, the Herfindahl index has a "ratio of sums of lognormals" distribution:

\[
HHI_m = \sum_{j=1}^{n} \left( \frac{\prod_{i=1}^{t} (1 + e_{ij})}{\sum_{j=1}^{n} \prod_{i=1}^{t} (1 + e_{ij})} \right)^2
\]

\[
= \frac{\sum_{j=1}^{n} \exp(2X_j)}{\left( \sum_{j=1}^{n} \exp(X_j) \right)^2}
\]

where the \( X_j \)'s are independent, identically normally distributed random variables.\(^{133}\)

The lack of a closed-form probability distribution function for finite sums of lognormals\(^{134}\) appears to impede systematic study of concentration in stochastic market models.\(^{135}\) For sufficiently large \( n \), however, a sum of \( n \) lognormals can be approximated by a single lognormal.\(^{136}\) This fact leads to several results concerning the variability of the HHI that are discussed here and proved in the Appendix.

The results in the Appendix provide estimates for finite sums of lognormals and, thus, the distribution of the observed HHI, given the number of firms \( n \), the variance of growth rates \( \sigma^2 \), and the duration of the model \( t \).

\(^{131}\) Scherer & Ross, supra note 126, at 142.

\(^{132}\) Throughout this Note, all logarithms and exponential functions use the natural base \( e = 2.718 \) unless otherwise specified.

\(^{133}\) See Appendix, Proposition 2.

\(^{134}\) This is a major open problem in statistics, and has been studied for some time by electrical engineers in a variety of signal processing contexts: "The characterization of the sum [of lognormals] is of importance in multihop scatter systems, log-normal shadowing environments, target detection in clutter, and the general problem of propagation through a turbulent medium." S C Schwartz & Y S Yeh, On the Distribution Function and Moments of Power Sums with Log-Normal Components, 61 Bell Sys Technical J. 1441, 1441-42 (1982).

\(^{135}\) See Ijiri & Simon, supra note 122, at 5 ("In all but the simplest cases closed solutions for the diffusion processes may not be available. As a safeguard against insufficient rigor leading to incorrect conclusions, we have done a considerable amount of simulation ... ").

\(^{136}\) See N.A. Marlow, A Normal Limit Theorem for Power Sums of Independent Random Variables, 46 Bell Sys. Technical J. 2081, 2082 (1967). This result, presented as an explanation of observed distributions in noise levels on trunk lines, does not appear to have reached the literature on the lognormal distribution of firm size.
These estimates confirm Scherer and Ross's observation that the concentration of a Gibrat market may be expected to increase steadily over time.

These results can be used to quantify measurement error in the HHI statistic by estimating confidence intervals for the HHI for a Gibrat market with \( n \) firms, as shown in Tables 3 and 4.\(^{137}\) Table 3 is suitable for estimating minimum \( p \) values to rebut the presumption of illegality. It demonstrates that the probability that observed HHI falls into the indicated interval is at most 90%. Table 4 is suitable for estimating maximum \( p \) values to show presumptive illegality. It demonstrates that the probability that observed HHI falls into the indicated interval is at least 90%.

IV. ANALYSIS OF THE HORIZONTAL MERGER GUIDELINES

As the courts continue to address the merger analysis suggested by the Horizontal Merger Guidelines, they will face increasingly complex findings of probabilistic economic facts. The presence of statistical error throughout the analysis calls for a balancing approach that assigns to each piece of evidence a weight commensurate with its statistical certainty. The Gibrat theory offers such a balancing framework.

Tables 3 and 4, as derived from the Gibrat theory, provide a basis for assessing the statistical certainty associated with market concentration, ease of entry, and concentration trends, and, thus, the evidentiary weight that should be assigned to each of these factors in a balancing approach to merger analysis. In general, the market concentration thresholds in the Horizontal Merger Guidelines establish a statistically significant presumption that a market is structurally anticompetitive. Ease of entry evidence can rebut this presumption by changing the parameters of the market model, thereby attenuating the inferential power of the market concentration statistics. A concentration trend, if steady and substantial, can provide relevant evidence about a market's competitive structure by forecasting the equilibrium concentration of the market. These findings are described in detail below.

A. Market Concentration

Areeda and Turner's leading antitrust treatise recognizes the case \( n=12 \) as a threshold for the minimum number of equivalent sellers in a market needed

\(^{137}\) For example, Table 3 indicates that in 100 simulations of a Gibrat market with \( n=50 \) firms and an expected HHI of 500, at most 90 would be expected to result in an HHI of between 143.0 and 1748.2, and so at least 10 runs would be expected to result in an HHI of less than 143.0 or greater than 1748.2. Table 4 indicates that at least 90 such simulations would be expected to result in an HHI of between 106.7 and 2342.2. This Note presents only approximate bounds indicating the widest and narrowest possible confidence intervals, based on an asymptotic estimation of the distribution of observed HHI values by the normal distribution.
to assure competitive pricing.\textsuperscript{138} Consider, therefore, a Gibrat market with

\textsuperscript{138} See 4 AREEDA & TURNER, supra note 27, ¶ 908, at 27 ("[T]here is a substantial consensus that independent pricing among sellers of a homogeneous product is likely to occur in a market with as few as

---

\textbf{Table 3. Estimated 90% Confidence Intervals for Observed HHI of Gibrat Markets Using Lower Bound on Variance}

<table>
<thead>
<tr>
<th>Number of Firms</th>
<th>Expected HHI</th>
<th>500</th>
<th>1000</th>
<th>1800</th>
<th>3000</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Min</td>
<td>Max</td>
<td>Min</td>
<td>Max</td>
<td>Min</td>
</tr>
<tr>
<td>12</td>
<td>—</td>
<td>—</td>
<td>934.9</td>
<td>1069.6</td>
<td>575.0</td>
</tr>
<tr>
<td>15</td>
<td>—</td>
<td>—</td>
<td>774.6</td>
<td>1291.0</td>
<td>253.7</td>
</tr>
<tr>
<td>20</td>
<td>500.0</td>
<td>500.0</td>
<td>502.1</td>
<td>1991.6</td>
<td>50.8</td>
</tr>
<tr>
<td>25</td>
<td>468.1</td>
<td>534.1</td>
<td>294.1</td>
<td>3400.3</td>
<td>7.9</td>
</tr>
<tr>
<td>30</td>
<td>417.4</td>
<td>599.0</td>
<td>159.3</td>
<td>6276.4</td>
<td>1.0</td>
</tr>
<tr>
<td>40</td>
<td>307.2</td>
<td>813.8</td>
<td>38.7</td>
<td>10000.0</td>
<td>—</td>
</tr>
<tr>
<td>50</td>
<td>210.4</td>
<td>1188.0</td>
<td>7.6</td>
<td>10000.0</td>
<td>—</td>
</tr>
</tbody>
</table>

\textbf{Table 4. Estimated 90% Confidence Intervals for Observed HHI of Gibrat Markets Using Upper Bound on Variance}

<table>
<thead>
<tr>
<th>Number of Firms</th>
<th>Expected HHI</th>
<th>500</th>
<th>1000</th>
<th>1800</th>
<th>3000</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Min</td>
<td>Max</td>
<td>Min</td>
<td>Max</td>
<td>Min</td>
</tr>
<tr>
<td>12</td>
<td>—</td>
<td>—</td>
<td>522.0</td>
<td>1915.5</td>
<td>164.3</td>
</tr>
<tr>
<td>15</td>
<td>—</td>
<td>—</td>
<td>351.4</td>
<td>2845.7</td>
<td>69.0</td>
</tr>
<tr>
<td>20</td>
<td>500.0</td>
<td>500.0</td>
<td>201.2</td>
<td>4969.6</td>
<td>13.4</td>
</tr>
<tr>
<td>25</td>
<td>299.0</td>
<td>836.0</td>
<td>112.6</td>
<td>8880.8</td>
<td>2.1</td>
</tr>
<tr>
<td>30</td>
<td>238.6</td>
<td>1047.4</td>
<td>59.8</td>
<td>10000.0</td>
<td>0.3</td>
</tr>
<tr>
<td>40</td>
<td>160.9</td>
<td>1553.6</td>
<td>14.2</td>
<td>10000.0</td>
<td>0.0</td>
</tr>
<tr>
<td>50</td>
<td>106.7</td>
<td>2342.2</td>
<td>2.8</td>
<td>10000.0</td>
<td>0.0</td>
</tr>
</tbody>
</table>
n=12 firms and expected HHI=1000. Table 4 indicates that the probability that the observed value of the HHI is between 522.0 and 1915.5 is at least 90%. In other words, a Gibrat market that is structurally competitive (n=12 and E(HHI)=1000) by DOJ standards has at most a 5% chance of appearing in the upper tail of this HHI distribution, i.e., having an observed HHI above 1915.5.

As this example illustrates, the Gibrat theory provides an independent justification for the selection of the threshold HHI value of 1800 as a trigger point for antitrust enforcement. In 1000 structurally competitive Gibrat markets, the observed HHI would be expected to exceed 1800 in at most 65 cases. To the extent that the Gibrat model is acceptable as an explanation for observed market concentration, an HHI value exceeding 1800 serves as prima facie evidence that a market is structurally anticompetitive.

In addition to providing a dynamic interpretation of the Horizontal Merger Guidelines, the framework may have the further salutary effect of focusing antitrust enforcement on strategic industries. In the Gibrat model, the expected HHI is an increasing function of the variance of the firms' annual growth rates, and high variance in growth rates appears to be associated with small businesses in highly innovative fields.

An interesting property of Tables 3 and 4 is that as the expected HHI increases, the confidence intervals for the observed HHI grow wider. This property of market concentration in the Gibrat model cautions against interpreting observed HHI values with excessive precision. The finding in Bon-Ton Stores that an increase in the HHI of 1679 points to 5074 was "off the charts" because it was "more than sixteen times" the DOJ threshold of 100 carries an unwarranted presumption of linearity and precision in the interpretation of observed HHI values. If the court's task is to evaluate the market concentration evidence as proof of market power, it should express its conclusions in terms of the inferential power of the HHI statistics instead of the raw statistics themselves. This Note provides a method by which it may do so.

10 to 12 equivalent sellers . . .

139. As Proposition 2 and Corollary 2 demonstrate, the parameters h=HHI/10000 and n fully specify the competitive structure of the market; any discrepancy between 10000h and the observed value of the HHI, 10000exp(H_n), is an artifact of chance. See Appendix.

140. An interval that includes all values within 1.645 standard errors of the expected value of a normally distributed random variable X is at least a 90% confidence interval for X. Corollary 2 gives an approximate upper bound of n^h+4h-S/n for the variance of Var(H_n), and so the standard error of H_n is at most (1.561)^2=3.951. Since E(H_n)=log(1.1)=2.0326, the interval -2.0326±(1.645)(3.951) is at least a 90% confidence interval for the normal approximation to H_n. Since 10000 exp (-2.0326±(1.645)(3.951)) =522.0, 1915.5, the interval (522.0, 1915.5) is at least a 90% confidence interval for the observed HHI of the Gibrat market.

141. In terms of statistical inference, an HHI value of 1800 corresponds to a p value of at most .065.

142. See, e.g., Evans, supra note 124, at 670–72.

B. Ease of Entry

The Scherer-Ross study, which considers the case \( n = 50 \), demonstrates that there is nothing sacrosanct about starting the Gibrat model with twelve equal-sized firms. Low barriers to entry and minimum efficient scale in a market can—and should—be reflected in the model by increasing \( n \), thereby allowing more firms into the competition for market share. The effect of such an increase in \( n \) is a corresponding increase in the variance of the observed HHI, which in turn reduces the inferential power of the market concentration statistic. This effect, illustrated in Tables 3 and 4, corresponds closely to the use of ease of entry evidence to rebut the presumption of illegality.

For example, a showing that the HHI for a market following a merger is 1915.5 may be rebutted by evidence that the market is more accurately described by a twenty-five-firm Gibrat model than a twelve-firm scenario. Referring to Table 3, when \( n = 25 \), an observed HHI of 1915.5 falls well within the 90% confidence interval and is found to correspond to a p-value of at least 0.191. In other words, the odds against the possibility that the observed high concentration (relative to the DOJ’s 1000 threshold) is the result of chance alone shorten from 19-1 to about 4-1. The rebuttal thereby has a direct and significant effect on the likelihood of successful prosecution on the merits.

This effect can be explained intuitively: A market that starts with twenty-five equal-sized firms is initially less concentrated than one that starts with twelve, and so the process by which it becomes concentrated to an HHI of 1915.5 tends to be longer and subject to greater uncertainty. This uncertainty exposes the HHI to such anomalies as the example illustrated in Table 1, where a significant change in the market shares of individual companies is not necessarily reflected in a corresponding change in the HHI. Consequently, an observed HHI of 1915.5 is much less informative about the

\[ \text{144. Note that increasing } n \text{ has the effect of describing the market as comprising two different sets of firms—existing firms that have attained positive current market share, and entrant firms that have not—but subject to the same dynamic growth assumptions. This corresponds to the condition of barrierless entry.} \]

\[ \text{A more sophisticated analysis would consider the market as a system of birth and death processes} \]

\[ \text{See } \text{IJIRI & SIMON, supra note 122, at 153–69. Such an analysis might also incorporate the theory of contestable markets. See generally Elizabeth E. Bailey & William J. Baumol, Deregulation and the Theory of Contestable Markets, 1 YALE J. ON REG. 111 (1984) (describing contestability theory in context of deregulation). Such analysis would be expected to yield results resembling those obtained here See SCHERER & ROSS, supra note 126, at 143 n.122 (“The [stochastic growth] distributions are sufficiently similar that it is difficult to find statistical tests distinguishing which of several alternative stochastic processes generated them.”).} \]

\[ \text{145. For a similar calculation, see supra note 140.} \]

\[ \text{146. Areeda and Turner contend that “once an industry has reached a particular degree of concentration... it is the present market structure that is critical, not the history of its getting there” Areeda & Turner, supra note 27, ¶ 914, at 82. This argument neglects the fact that in some industries, a trend toward concentration may be volatile, reducing confidence in the precision of a market snapshot. See supra note 7.} \]
underlying conditions of a twenty-five-firm market than about those of a twelve-firm market.

The defendant in a horizontal merger case has various options at trial for making a claim about the value of $n$. First, the number $n_e$ of existing firms could serve as a crude lower bound for $n$. Second, the defendant could make an economic argument that it would be reasonably likely that $n-n_e$ firms, exercising proportionate market power, would enter the market before a dominant firm could sustain a significant price increase. Third, the defendant could make an economic determination concerning the number of new minimum-efficient-scale firms that could be accommodated by the market. Finally, the defendant could forecast future market structure from a historical pattern of market entry.

The fair evaluation of ease of entry arguments requires that defendants quantify their claims of low entry barriers and that judges balance these claims against observed market concentration. As one observer has warned:

If a finding of "ease of entry" or "low entry barriers" is sufficient to overcome prima facie illegality under Section 7, those terms of art must be given substance and quantitative meaning. Otherwise, merger inquiries will degenerate into a battle of the experts over obscure and subjective evaluations of "low" and "high" barriers. Judges will have the discretion to rewrite the antitrust laws to their liking, and litigants will go on sprees of forum shopping.

The DOJ assumes an entry period of two years and a price increase of five percent. See Department of Justice 1992 Horizontal Merger Guidelines, 57 Fed. Reg. 41,552, 41,562 (1992). This baseline for market definition has been criticized as "[construing] the relevant market too broadly." Gene C. Schaerr, Note, The Cellophane Fallacy and the Justice Department's Guidelines for Horizontal Mergers, 94 YALE L.J. 670, 670–71 (1985). The framework presented here is independent of market definition methodologies and could subsume Schaerr's proposed reforms.

The fair evaluation of ease of entry arguments requires that defendants quantify their claims of low entry barriers and that judges balance these claims against observed market concentration. As one observer has warned:

If a finding of "ease of entry" or "low entry barriers" is sufficient to overcome prima facie illegality under Section 7, those terms of art must be given substance and quantitative meaning. Otherwise, merger inquiries will degenerate into a battle of the experts over obscure and subjective evaluations of "low" and "high" barriers. Judges will have the discretion to rewrite the antitrust laws to their liking, and litigants will go on sprees of forum shopping.

The fair evaluation of ease of entry arguments requires that defendants quantify their claims of low entry barriers and that judges balance these claims against observed market concentration. As one observer has warned:

If a finding of "ease of entry" or "low entry barriers" is sufficient to overcome prima facie illegality under Section 7, those terms of art must be given substance and quantitative meaning. Otherwise, merger inquiries will degenerate into a battle of the experts over obscure and subjective evaluations of "low" and "high" barriers. Judges will have the discretion to rewrite the antitrust laws to their liking, and litigants will go on sprees of forum shopping.

See Hovenkamp, supra note 19, § 12.4c, at 474–76 (enumerating alternative forms of ease of entry evidence).

See, e.g., United States v. Country Lake Foods, Inc., 754 F. Supp. 669, 672 (D. Minn. 1990) (rejecting government's claim that milk processing market consisted of only eight dairies in Minneapolis-St. Paul MSA). A more refined estimate would be the reciprocal of the market share of the median firm. This measure is based on empirical findings that minimum efficient plant scales are "significantly correlated with . . . the ratio of sales of plants at the midpoint of industry plant size distributions to total industry sales." Scherer & Ross, supra note 126, at 425.

See, e.g., Country Lake Foods, 754 F. Supp. at 672–73 (discussing testimony of six distant dairies that they would enter Minneapolis-St. Paul market in response to "nontransitory increase" in milk prices); United States v. Baker Hughes Inc., 731 F. Supp. 3, 11 (D.D.C. 1990) ("It appears likely to the Court, however, that [in the event of merger] . . . one or two . . . companies will enter successfully sometime in the future, because major United States customers, who are quite sophisticated and financially strong, will insist on receiving alternate bids.").
The framework in this Note fills the implied void, bringing substance and quantitative meaning to the ease of entry rebuttal by making it commensurable with the presumption of illegality based on HHI statistics.

C. Concentration Trends

Breaking with precedent,\textsuperscript{153} William F. Baxter, the chief architect of the 1982 DOJ Horizontal Merger Guidelines, stated in 1983 that as a matter of policy, a trend toward market concentration would not be considered as a factor in analyzing horizontal mergers.\textsuperscript{154} The deletion of concentration trends from the DOJ/FTC Horizontal Merger Guidelines\textsuperscript{155} has won the approval of some leading commentators.\textsuperscript{156} On the other hand, the case law indicates that trends toward market deconcentration will continue to be considered in weighing the presumption of illegality based on HHI statistics.\textsuperscript{157}

The Gibrat theory indicates that trends toward both concentration and deconcentration are relevant in evaluating an observed HHI statistic. The law of regression toward the mean\textsuperscript{158} implies that trends in the observed HHI are more likely than not to be directed toward the expected HHI determined by the market's competitive structure. In particular, a defendant's contention that an observed HHI of 1915.5 was a chance outlier from an expected HHI of 1000 would be more consistent with a long-term trend toward deconcentration than with one toward concentration. Thus, as in \textit{Kraft General Foods,},\textsuperscript{159} a trend toward deconcentration may constitute evidence in rebuttal of HHI-based prima facie illegality. As the Gibrat framework makes clear, such a rebuttal amounts to an assertion that the trend line is pointing away from a temporarily high level of concentration and toward the underlying competitive structure of the market.

Despite the relevance of concentration trends within the Gibrat framework, it is clear that trend data should be evaluated with care and even skepticism.


\textsuperscript{154} See Baxter, \textit{supra} note 96, at 630.

\textsuperscript{155} The current NAAG guidelines continue to allow an attorney general to institute actions based on trends toward concentration. \textit{See Horizontal Merger Guidelines of the National Association of Attorneys General,} 4 Trade Reg. Rep. (CCH) \textsuperscript{156} 13,406, at 21,203 n.34 (Mar. 10, 1987).


\textsuperscript{158} The law of regression toward the mean states that probabilistic outcomes that are above the expected value are more likely than not to be followed by lower outcomes. See, e.g., \textit{David Freedman ET AL., STATISTICS} ch. 10 (1991). Thus an unusually high concentration level that was due to chance would probably be followed by a steady and substantial trend toward deconcentration.

\textsuperscript{159} 926 F. Supp. at 363; see \textit{supra} text accompanying notes 112–16.
At best, a statistical argument based on regression toward the mean can only test an alternative hypothesis, leaving the court to weigh competing explanations of observed market concentration. To have inferential power, a concentration trend should be both steady and substantial over a long time period.

This Note has focused on only two of the factors that may rebut a prima facie claim based on the HHI thresholds: ease of entry and concentration trends. These factors were chosen because in all but one of the cases examined in Table 2 in which the presumptive illegality was successfully rebutted, one or both of these factors was dispositive. Further work will be needed to determine the implications of the Gibrat theory for other factors discussed in the Horizontal Merger Guidelines.

V. CASE STUDY: COUNTRY LAKE FOODS

United States v. Country Lake Foods Inc. clearly illustrates the role that this Note's framework can have in balancing structural evidence in horizontal merger cases. In 1990, the Department of Justice sought to enjoin the acquisition by Country Lake Foods of a competing dairy, Superior-Dairy Fresh Milk, as a violation of section 7 of the Clayton Act. The complaint defined the relevant market as fluid milk suppliers in the Minneapolis-St. Paul metropolitan statistical area (MSP/MSA), consisting of eight local dairies. In this market, Country Lake and Superior were the second- and third-largest sellers, with shares of 18.2% and 17.8%, respectively. The acquisition would increase the HHI of the market from 2186 to 2832. These market concentration statistics triggered the government's motion for injunctive relief.

The defendants countered with an alternative geographic market definition that included dairies within a 350-mile radius of Minneapolis-St. Paul. They argued that modern milk processing and transportation methods and a relative lack of brand name differentiation had expanded the relevant geographic market. The district court agreed, persuaded by testimony by

160. The remaining case was FTC v. Freeman Hospital, 911 F. Supp. 1213, 1222 (W.D. Mo. 1995), aff'd, 69 F.3d 260 (8th Cir. 1995). See supra notes 71, 91.
163. See id. at 670.
164. See id. at 671.
165. See id. at 673.
166. See id.
167. See id. at 671 ("[T]he proposed acquisition . . . may substantially lessen competition in the relevant product and geographic markets.").
168. See id. at 672 n.3.
169. See id. at 672-73.
several distant dairies that it was likely that they would begin selling milk in
Minneapolis-St. Paul if it became profitable for them. Specifically, the
court found that five such dairies would definitely or probably enter the market
to fill orders from grocers in the MSP/MSA provided that they could sell from
excess capacity at a profitable price. Another dairy specified that it would
begin selling in the MSP/MSA if the price of milk rose by seven percent.

Although none of the six distant dairies was a significant competitor in the
Minneapolis-St. Paul market in 1990, the court viewed them as a
safeguard against any market power that might be created by the merger.
Noting that distant dairies could profitably serve the entire MSP/MSA in the
event of a five percent price increase, the court concluded that consumers in
Minneapolis-St. Paul could “practically turn to dairies outside the MSP/MSA
for fluid milk should a price increase occur based on a cartel among dairies in
the MSP/MSA.” Consequently, the government failed to meet its burden
of defining the relevant geographic market, and the court turned to
nonstructural factors in analyzing, and ultimately allowing, the proposed
acquisition.

The simplicity and reasonableness of this economic argument disguised the
extent to which the court relied on unfounded conjectures in dismissing the
government’s structural claims. The court defined the geographic market to
include both the eight MSP/MSA dairies and the six distant dairies, but then
envisioned a postacquisition market segmented into two submarkets:
MSP/MSA dairies, which may be inclined to cartelize, and distant dairies,
which can be relied upon to prevent cartelization. The court cannot have it
both ways. If both groups of dairies were really in the same market, then all
firms would benefit from cartelize and monopoly rents. The proper inquiry
should be whether the overall structure of the postacquisition market presented
a substantial risk of anticompetitive firm behavior. For this inquiry, the court
would have to have heeded the government’s HHI statistics. Since the distant
dairies possessed negligible aggregate market share in Minneapolis-St. Paul,
the HHI of the MSA eight-dairy submarket was a good estimate of the HHI
of the overall fourteen-dairy market. Instead, the court threw the market
concentration statistics out with the geographic market definition.

Even more troubling was the court’s failure to quantify the force of its
market entry argument. Would the court’s reasoning have applied if Country
Lake Foods and Superior each had held half of the MSP/MSA market and

170. See id. at 672.
171. See id. at 672-73.
172. See id. at 673.
173. The top four milk producers in the MSP/MSA had an aggregate market share of 90.2% following
the proposed acquisition. See id.
174. Id.
175. See id.
176. See id. at 673-74.
were merging to monopoly? What if Minneapolis-St. Paul consumers had had only one distant dairy waiting in the wings? Short of these extremes, how many points of excess HHI may a potential entrant firm excuse? The court is silent on these critical issues.

By showing how market concentration statistics and ease of entry arguments may be balanced quantitatively, the analytic framework in this Note provides tentative answers to these questions. Although these answers are neither precise nor dispositive, they do supply a rationale for distinguishing the court's decision from the unattractive hypotheticals above.

As Table 4 indicates, the postacquisition HHI of 2832 still has evidentiary power in a fifteen-firm market (p<.05 for HHI=2845.7), but not in a twenty-five-firm market. To ensure a competitive postacquisition market, the court should therefore require that the defendants find approximately eleven more distant dairies ready to serve the MSP/MSA. Viewed from the stochastic market perspective, the involvement of more firms would be evidence that the level of concentration observed in 1990 was more likely to have been the result of chance over time than a consequence of market power. More firms in the market would also increase the likelihood that some distant dairies will, by chance, break out of the fringe and achieve substantial shares, thereby reducing future concentration.

Faced with ease of entry evidence, a court wishing to apply the Gibrat analytical framework should first inquire whether the Gibrat assumption of size-independent growth rates is appropriate for the market. Second, the court should ensure that the entrant firms face no barriers to entry, so that an increase in \( n \) accurately accounts for their possible entry. Third, the court should determine whether the increase in \( n \) is sufficient to create statistical uncertainty about the competitive structure of the market.

Even if the Gibrat assumptions are inappropriate for a particular market, a court should recognize that the determination of anticompetitive effect requires that it adopt some set of dynamic market assumptions. Those assumptions should be explicit in the trial record, thereby limiting the precedential value of each case to markets with similar dynamic properties. In this way, the case law on horizontal mergers can avoid turning ease of entry into a trump and, instead, can develop coherently according to rational economic principles.

Antitrust jurisprudence has been remarkably responsive to the economic analysis embodied in the 1982, 1984, and 1992 DOJ Horizontal Merger Guidelines. By recognizing and expounding the dynamic features of that analysis, the courts can make substantial progress toward a unified horizontal

---

177. Because Tables 3 and 4 present approximate confidence intervals for observed HHI, this discussion should not be taken to suggest dispositive thresholds for the number of firms, but only the overall structure of analysis within the Gibrat framework. See supra note 137. In all cases, greater precision may be obtained through more fact-specific dynamic modeling of market structure.
merger doctrine. Extrapolating from the arc of economic sophistication between Von's Grocery and Kraft General Foods, there is every reason to believe that they will.

VI. CONCLUSION

This Note has presented a framework for drawing statistical inferences from the most common forms of evidence presented in recent horizontal merger cases. By balancing the issues of market concentration, ease of entry, and concentration trends quantitatively, this framework can provide guidance in weighing evidence in a field where little or no case law exists.

More importantly, this Note has provided a new and useful interpretation of the horizontal merger doctrine. As a probabilistic treatment, it acknowledges the uncertainty inherent in using the HHI as a proxy for market power. High HHI values are accepted as evidence of market power not because markets satisfy strict, static, microeconomic assumptions, but because they are much more likely to occur as the result of anticompetitive market conditions than by pure chance. Low barriers to entry rebut this evidence, not because ease of entry always trumps concentration, but because the entry of more firms makes concentration by chance more likely. Concentration trends are relevant not because the history of a market will repeat itself, but because trend lines usually point toward equilibria, obeying the law of regression toward the mean. The perspective of “antitrust by chance” thus appears to be founded not only in advanced statistical methods, but also in common sense.

180. See, e.g., Kwoka, supra note 20, at 924–26.
181. See Hsu, supra note 3, at 75.
182. See United States v. Philadelphia Nat'l Bank, 374 U.S. 321, 367 (1963) (ruling that “trend toward concentration” would make it necessary to prevent first merger in 40-bank market before “30 more Philadelphia banks were absorbed”).
The mathematical propositions in this Note rely upon the following result:¹⁸³

**PROPOSITION 1.** (Marlow)

Suppose that \((X_i)\) is a sequence of mutually independent, identically distributed random variables. Then, as \(n \to \infty\), the distribution of

\[
P_n = 10 \log_{10}(10^{X_1/10} + \cdots + 10^{X_n/10})
\]

is asymptotically normal with mean

\[
10 \log_{10}(n \cdot E(10^{X_i/10}))
\]

and variance

\[
\frac{100 \text{Var}(10^{X_i/10})}{n (\log 10 \cdot E(10^{X_i/10}))^2}.
\]

Marlow proved this result in order to explain empirical distributions of noise levels when measured on a decibel scale.¹⁸⁴ A change from base 10 to base \(e\) is necessary for the purposes of this Note. The substitutions \(X_i' = \frac{X_i \log 10}{10}, i = 1, \ldots, n\) yield:

**COROLLARY 1.**

Suppose that \((X_i)\) is a sequence of mutually independent, identically distributed random variables. Then, as \(n \to \infty\), the distribution of

\[
P_n = \log \sum_{i=1}^{n} \exp X_i
\]

is asymptotically normal with mean

\[
\log(n \cdot E(\exp X_1))
\]

and variance

\[
\frac{\text{Var}(\exp X_1)}{n (E(\exp X_1))^2}.
\]

¹⁸³. See Marlow, supra note 136, at 2086.
¹⁸⁴. See id. at 2082.
PROPOSITION 2.

For \( i=1, \ldots, t \) and \( j=1, \ldots, n \), let \( (Z_n)_j=\log(1+e_j) \) be a sequence of independent random variables normally distributed with mean \( z \) and variance \( s^2 \). Then, as \( n \to \infty \), the distribution of

\[
H_n = \log \sum_{j=1}^{n} \left( \frac{\prod_{i=1}^{t} (1 + e_{ij})}{\sum_{j=1}^{n} \prod_{i=1}^{t} (1 + e_{ij})} \right)^2
\]

is asymptotically normal with mean

\[
tS^2 - \log n
\]

and variance \( V \), where

\[
\exp(4tS^2) + 4\exp(tS^2) - 5 - 4(\exp(4tS^2) - 1)^{1/2}(\exp(tS^2) - 1)^{1/2} \leq V
\]

\[
\leq \exp(4tS^2) + 4\exp(tS^2) - 5 \frac{n}{n}.
\]

PROOF.

Using the properties of sums of independent normal variables, the product

\[
\prod_{i=1}^{t} (1 + e_{ij}) = \exp \sum_{i=1}^{t} \log (1 + e_{ij}) = \exp \sum_{i=1}^{t} Z_{ij}
\]

for \( j=1, \ldots, n \), can be rewritten as the lognormal \( \exp X_{ij} \), where each \( X_{ij} \) is an independent random variable normally distributed with mean \( tz \) and variance \( tS^2 \). This gives

\[
H_n = \log \sum_{j=1}^{n} \left( \frac{\prod_{i=1}^{t} (1 + e_{ij})}{\sum_{j=1}^{n} \prod_{i=1}^{t} (1 + e_{ij})} \right)^2
\]

\[
= \log \sum_{j=1}^{n} \exp (2X_{ij}) - 2\log \sum_{j=1}^{n} \exp X_{ij} \quad (1)
\]
The expected value and variance of the lognormal \( \exp X \), where \( X \) is normally distributed with mean \( \bar{X} \) and variance \( s^2 \), are given by

\[
E(\exp X) = \exp(\bar{X} + s^2/2)
\]

and

\[
\text{Var}(\exp X) = \exp(2\bar{X} + s^2)(\exp(s^2) - 1).
\]

Thus, by Corollary 1, as \( n \to \infty \), the first term in (1) is asymptotically normal with mean

\[
\log n + 2t\bar{Z} + ts^2
\]

and variance

\[
\frac{\exp(4ts^2) - 1}{n};
\]

and the second term in (1) is asymptotically normal with mean

\[
2\log n + 2t\bar{Z} + ts^2
\]

and variance

\[
\frac{4(\exp(ts^2) - 1)}{n}.
\]

The distributions of these two terms have nonnegative correlation, since for every \( y \),

\[
P\left[ \sum_{j=1}^{n} \exp(2X_j) > y \middle| \sum_{j=1}^{n} \exp(X_j) > x \right]
\]

is nondecreasing in \( x \) (because of the convexity of the exponential function).\(^{185}\) The result now follows from the identity

\[
\bar{X} - \bar{Y} = \bar{X} - \bar{Y}
\]

and the inequality

\[
\text{Var}(X - Y) = \text{Var}(X) + \text{Var}(Y) - 2\text{Cov}(X,Y)
= \text{Var}(X) + \text{Var}(Y) - 2\text{Corr}(X,Y)(\text{Var}(X)\text{Var}(Y))^{1/2}
\geq \text{Var}(X) + \text{Var}(Y) - 2(\text{Var}(X)\text{Var}(Y))^{1/2}.
\]

---

\(^{185}\) For various sufficient conditions for nonnegative correlation, see Kumar Jogdeo, *Dependence, Concepts of*, in *2 Encyclopedia of Statistical Sciences* 324, 326 (Samuel Kotz & Norman L. Johnson eds., 1982).
COROLLARY 2.

Let \((Z_0)\) and \(H_n\) be as in Proposition 2, and suppose that parameters \(i, n, z\) and \(S\) are chosen so that as \(n \to \infty\),

\[ E(H_n) = \log h \]

uniformly for some constant \(h, 0 < h < 1\). Then

\[ \text{Var}(H_n) = \nu, \]

where

\[ h^4n^3 + 4h - \frac{5}{n} - \frac{4(h^4n^4 - 1)^{1/2}(hn - 1)^{1/2}}{n} \leq \nu \leq h^4n^3 + 4h - \frac{5}{n}. \]