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Beyond Document Retrieval toward Information Retrieval

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Beyond Document Retrieval
Toward Information Retrieval

Electronic data processing systems are being seriously considered for use in legal research, and a veritable flood of articles has appeared in various legal periodicals speculating about the use to which such systems can be put in legal context. The general tone of these articles is understandable, for "we are all groping to achieve a better understanding of the relation of these developments in communication technology and law, and it is to be expected that discussion will be vague and general at the outset." As one step in the direction of more explicitly defining the role of computers in the law, Professor Allen presents a specific proposal for improving the drafting of legal documents so that some of the logical analysis of the contents can be performed automatically. He then applies this proposal to sample sections of the federal estate tax.

This Article is not easy reading, and a casual glance at the text may unfortunately discourage all but a few from even beginning it. But the steps as Professor Allen has drawn them are gradual and are not difficult to follow. Should you decide to work through the Article, we think that your knowledge and appreciation of this potential revolution in legal methods will be greatly enhanced.

Layman E. Allen*

*Assistant Professor of Law, Yale Law School. This Article is adapted from a paper presented at the Conference on Tax Administrative and Tax Policy Implications of Electronic Data Processing at Harvard University from April 13 to 15, 1961. In the preparation of this Article, the author had the benefit of many helpful discussions with members of the ALL Project (Accelerated Learning of Logic) staff of the Yale Law School: Robin B. S. Brooks, James W. Dickoff, and Patricia A. Jones. Mary E. Caldwell also commented upon and made suggestions about an earlier draft. Also, parts of the Article were written while the author was a fellow at the Center for Advanced Study in the Behavior Sciences. The author is indebted to all of them, but takes full responsibility for what is asserted here.
The thesis set forth here is a simple one:

If

1. the written materials used in the tax field are more systematically drafted,

then

2. human beings will be able to "read" and "work with" those materials "better," and

3. automatic devices will be able to "read" and "work with" those materials "better."

In the first part of this Article, a tentative description of one method of organizing the written materials in the tax field is presented. In the second part, sample sections of the gross estate provisions of the Federal Estate Tax of the Internal Revenue Code are reorganized and presented as an illustration of the method of organization outlined in the first part. Finally, in the third part, there are discussed and illustrated some of the kinds of operations that mechanical and electronic devices will be able to perform with respect to materials that are organized along the suggested lines.

I. MORE SYSTEMATIC ORGANIZATION OF WRITTEN TAX MATERIALS

A. SENTENCES, SENTENCE-PATTERNS, AND STATEMENTS

S1 It is useful for lawyers to learn about prescriptive language systems.

S2 It is useful for lawyers to learn about prescriptive language systems.

S3 The first item in this section is an occurrence of a given sentence-pattern. The second item in this section is a second occurrence of the same sentence-pattern.

S4 An occurrence of a sentence-pattern is a sentence. Therefore, the first item in this section is a sentence. Similarly, the second item in this section is a sentence. Although the first and second items in this section are different sentences, they are occurrences of the same sentence-pattern.

S5 Sentence-patterns may be written, and the result of writing a sentence-pattern is a written sentence. Similarly, sentence-
patterns may be spoken, and the result of speaking a sentence-pattern is a spoken sentence.

S 6 For our purposes, the word 'statement' will be used to refer to the set of one or more sentences that immediately follow a given S#. Thus, the sentence immediately following 'S1' above will be referred to as the first statement of this section. Similarly, the three sentences immediately following 'S4' above will be referred to as the fourth statement of this section.

S 7 Now that we have distinguished between sentences, sentence-patterns, and statements, it will be useful to illustrate the representation of sentences (and thus statements) in a logical notation.

B. REPRESENTING A SENTENCE—ELEMENTARY REPRESENTATIONS

S 1 It is possible to represent a given sentence. If we do represent a given sentence, then we show that sentence's logical form.

S 2 1. It is possible to represent a given sentence, and
   2. if
      a. we do represent a given sentence,
      then
      b. we show that sentence's logical form.

S 3 Thus, a representation of a given sentence shows that sentence's logical form. It may be somewhat misleading to talk about the logical form of a given sentence, because some sentences have more than one logical form. A representation of a given sentence shows one of the logical forms of that sentence. Therefore, when we represent a sentence, we show one of its logical forms.

S 4 The second statement in this section is an example of a sentence that has more than one logical form. Therefore, the second statement can be represented in more than one way. 'S2' is one representation of the second statement in this section. However, 'S2' does not show anything about the logical relationships between the ideas expressed by the constituent sentences of the second statement.

S 5 If
   1. a representation of a given sentence does not show
anything about the logical relationships between the ideas expressed by the constituent sentences of that sentence,
then
2. we call that representation an elementary representation of that sentence.

S 6 Since 'S2' does not show anything about the logical relationships between the ideas expressed by the constituent sentences of the second statement, 'S2' is an elementary representation of the second statement. Similarly 'S3' is an elementary representation of the third statement in this section, and 'S5' is an elementary representation of the fifth statement in this section. Every sentence has an elementary representation.

S 7 There are some sentences that are, themselves, constituents of other sentences; many of these constituent sentences will also have elementary representations. For example, the first constituent sentence of the fifth statement in this section is just like the following sentence: "A representation of a given sentence does not show anything about the logical relationships between the ideas expressed by the constituent sentences of that sentence." 'S5.1' is an elementary representation of this first constituent sentence. Similarly, 'S5.2' is an elementary representation of the second constituent sentence of the fifth statement in this section.

S 8 The first constituent sentence of the second statement in this section is just like the following sentence: "It is possible to represent a given sentence," and 'S2.1' is an elementary representation of it. The second constituent sentence of the second statement in this section is just like the following sentence: "We do represent a given sentence," and 'S2.2a' is an elementary representation of it. (Look back at the second statement in this section to see why its second constituent sentence is represented by 'S2.2a', rather than by 'S2.2'.) The third constituent sentence of the second statement in this section has 'S2.2b' as its elementary representation.

S 9 As you might expect, 'S2' is an elementary representation of the entire second statement in this section, and the following is a partial representation of it: S2.1 and if S2.2a then S2.2b.
Now that we know how to show one logical form of a given sentence by means of an elementary representation, it will be useful to consider the following nonelementary representations of sentences:

1. Implication-representations;
2. disjunction-representations;
3. conjunction-representations;
4. equivalence-representations;
5. negation-representations.

C. REPRESENTING A SENTENCE—NONELEMENTARY REPRESENTATIONS

1. Implication-Representations

<table>
<thead>
<tr>
<th>Antecedent</th>
<th>Consequent</th>
</tr>
</thead>
<tbody>
<tr>
<td>S 1 If a lawyer learns some modern logic, then b. that lawyer will probably be a better legal draftsman than he would otherwise be.</td>
<td></td>
</tr>
<tr>
<td>2. $\rightarrow$ a. a lawyer learns some modern logic, $\rightarrow$ b. that lawyer will probably be a better legal draftsman than he would otherwise be.</td>
<td></td>
</tr>
<tr>
<td>3. If S1.1a then S1.1b</td>
<td></td>
</tr>
<tr>
<td>4. $\rightarrow$ S1.1a $\rightarrow$ S1.1b</td>
<td></td>
</tr>
</tbody>
</table>

By comparing S1.1 and S1.2, we see that 'If' is represented by '→', and 'then' is represented by '→'.

Furthermore, by comparing S1.1 and S1.3, we see that the first constituent sentence of S1.1 (the antecedent) is represented by 'S1.1a', and the second constituent sentence of S1.1 (the consequent) is represented by 'S1.1b'.

S 2 By comparing S1.1 and S1.2, we see that 'If' is represented by '→', and 'then' is represented by '→'.

S 3 Furthermore, by comparing S1.1 and S1.3, we see that the first constituent sentence of S1.1 (the antecedent) is represented by 'S1.1a', and the second constituent sentence of S1.1 (the consequent) is represented by 'S1.1b'.
The antecedent of an implication-sentence is always a sentence (as in S1.1); similarly, the consequent of an implication-sentence is always a sentence. However, the antecedent of a partial implication-representation may sometimes be a sentence (as in S1.2), it may sometimes be a representation (as in S1.3), and it may sometimes be a combination of both sentences and representations. Similarly, the consequent of a partial implication-representation may sometimes be a sentence, sometimes a representation, and sometimes a combination of both. But the antecedent of a complete implication-representation will always be a representation (as in S1.4), as will the consequent. [Hereafter, because most of the references will be to complete representations, when the word 'representation' is used, it will refer to a complete representation; if the reference is to a partial representation, the words 'partial representation' will be used.]

In an implication-sentence, two sentences are connected by 'if-then' (as in S1.1), while in a partial implication-representation, either

1. two sentences (as in S1.2), or combinations of sentences and representations, are connected by '→', or
2. two representations of sentences (as in S1.3), or combinations of sentences and representations, are connected by 'if-then'.

However, in an implication-representation, two representations of sentences are connected by '→' (as in S1.4).

If

1. one sentence is preceded by the word 'if' and followed by the word 'then', and
2. that 'then' is followed by a second sentence, then
3. we will say that
   a. the resulting sentence is an implication-sentence,
   b. the idea expressed by that first sentence implies the idea expressed by that second sentence, and
   c. the implication relationship exists between the idea expressed by that first sentence and the idea expressed by that second sentence, and
4. we will also say that
   a. every nonelementary representation of that resulting sentence is an implication-representation,
b. every representation of that first sentence implies every representation of that second sentence, and

c. the implication relationship exists between every representation of that first sentence and every representation of that second sentence.

Thus, we will say that the sentence represented by 'S1.1' is an implication-sentence, that the representation represented by 'S1.4' (i.e., 'implies S1.1a implies S1.1b') is an implication-representation, that S1.1a implies S1.1b, and that the implication relationship exists between S1.1a and S1.1b.

S7 If and only if
1. a given sentence is comprised of an 'if-then' connecting two constituent sentences of that given sentence,
then
2. we call that given sentence an implication-sentence,
3. that given sentence has a representation that begins with a 'implies' and contains a 'implies', and
4. that representation is an implication-representation.

S8 You have probably already observed how much 'implies' and 'implies' look like the tail and head of an arrow, respectively. Hereafter, we may sometimes refer to 'implies' as the tail, to 'implies' as the head, and to the combination of both, namely 'implies', as the arrow.

S9 If
1. a given sentence contains more than one 'if-then',
then
2. if
   a. we wish to represent that given sentence,
   then
      b. there is a representation containing two arrows that may be used to represent that given sentence.

S10 The ninth statement in this section can be represented in a variety of ways:
1. 'S9' is an elementary representation of it.
2. 'implies S9.1implies S9.2' is a one-arrow implication-representation of it.
3. 'implies S9.1implies S9.2implies S9.2aimplies S9.2b' is a two-arrow implication-representation of it.

The antecedent of 'implies S9.1implies S9.2implies S9.2b' is 'S9.1', while the consequent is 'implies S9.2aimplies S9.2b'. The antecedent of the consequent of 'implies S9.1
S 11 1. A representation of a given sentence is a nonelementary representation of that sentence if and only if
2. that representation shows something about the logical relationship between the idea expressed by a constituent sentence of that given sentence and the idea expressed by some other constituent of that given sentence.

S 12 Recall also that it was asserted above that implication-representations are nonelementary representations. This leads one to wonder what logical relationship is shown in an implication-representation between
1. the idea expressed by a constituent sentence of the given sentence which that implication-representation represents, and
2. the idea expressed by some other constituent of that given sentence.

Once we know that implication is a logical relationship, it is evident that what is shown in a given implication-representation is that the logical relationship of implication exists between the idea expressed by the antecedent and the idea expressed by the consequent of the implication-sentence which that given implication-representation represents. The words in a given implication-sentence that denote the logical relationship of implication between the idea expressed by the antecedent and the idea expressed by the consequent of that given implication-sentence are the words 'if-then'.

S 13 In sentences that have nonelementary representations, some constituents are sentence-constituents, while other constituents are nonsentence-constituents. In an implication-sentence, the antecedent and the consequent are sentence-constituents, while the 'if-then' is a nonsentence-constituent.

S 14 At the level of analysis being considered in this paper, all of the representations of nonsentence-constituents of nonelementary representations of sentences are called connectives. Therefore, ‘\( \rightarrow \)’ is a connective.

S 15 In representing sentences that have nonelementary representations, representations of nonsentence-constituents are clearly distinguished (with some exceptions) from represen-
tations of sentence-constituents. For example, the antecedent and consequent of a given implication-sentence are represented by an alpha-numeric expression, while the 'if-then' of that given sentence is represented by an arrow. Thus, in the implication-representation of the ninth statement in this section presented in S10.2, the antecedent was represented by 'S9.1', the consequent was represented by 'S9.2', and the 'if-then' was represented by '--- >'.

S 16 Now that we have learned how to show the logical form of an implication-sentence by means of an implication-representation, it is appropriate to consider the next set of non-elementary representations, the disjunction-representations.

### 2. Disjunction-Representations

<table>
<thead>
<tr>
<th>First Disjunct</th>
<th>Second Disjunct</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>S1.1</strong></td>
<td><strong>S1.1b</strong></td>
</tr>
<tr>
<td>a. A sentence that contains two 'if-then's can be represented by a two-arrow representation</td>
<td>b. that sentence can be represented by a one-arrow representation.</td>
</tr>
<tr>
<td>2. { a. A sentence that contains two 'if-then's can be represented by a two-arrow representation</td>
<td>b. that sentence can be represented by a one-arrow representation.</td>
</tr>
<tr>
<td>3. S1.1a or S1.1b</td>
<td></td>
</tr>
<tr>
<td>4. { S1.1a }</td>
<td></td>
</tr>
<tr>
<td>{ S1.1b }</td>
<td></td>
</tr>
</tbody>
</table>

Disjunction-sentence

Partial disjunction-representation

Partial disjunction-representation

Disjunction-representation
S 2 By comparing S1.1 and S1.2, we see that 'or' is represented by '{ }'.

S 3 Furthermore, by comparing S1.1 and S1.3, we see that the first constituent sentence of S1.1 (the first disjunct) is represented by 'S1.1a', and the second constituent sentence of S1.1 (the second disjunct) is represented by 'S1.1b'.

S 4 The disjuncts of a disjunction-sentence are always sentences (as in S1.1). The disjuncts of a partial disjunction-representation may sometimes be sentences (as in S1.2), may sometimes be representations (as in S1.3), and may sometimes be combinations of both sentences and representations; but the disjuncts of a disjunction-representation will always be representations (as in S1.4).

S 5 In a disjunction-sentence, two sentences are connected by 'or' (as in S1.1), while in a partial disjunction-representation, either

1. two sentences (as in S1.2), or combinations of sentences and representations, are connected by '{ }', or
2. two representations of sentences (as in S1.3), or combinations of sentences and representations, are connected by 'or'.

However, in a disjunction-representation, two representations of sentences are connected by '{ }' (as in S1.4).

S 6 If

1. a given sentence is comprised of two constituent sentences linked by the word 'or',
then
2. we will say that
   a. the given sentence is a disjunction-sentence, and
   b. the idea expressed by that first constituent sentence is disjoined with the idea expressed by that second constituent sentence, i.e., the disjunction relationship exists between those ideas, and
3. we will also say that
a. every nonelementary representation of that given sentence is a disjunction-representation, and
b. every representation of that first constituent sentence is disjoined with every representation of that second constituent sentence, i.e., the disjunction relationship exists between those representations.

Thus, we will say that the sentence represented by 'S1.1' is a disjunction-sentence, that the representation represented by 'S1.4' ('\{S1.1a\} \{S1.1b\}') is a disjunction-representation, and that the disjunction relationship exists between S1.1a and S1.1b.

S7 If
1. a given sentence is a disjunction-sentence,
then
2. that given sentence has a representation that contains a
   \{'\}\',

S8 If

1. a representation of a given sentence contains a \{'\}\',

then
2. a. that representation is a disjunction-representation, or
   b. that representation contains a disjunction-representation.

S9 The following representation of the eighth statement in this section is an example of a representation that contains a disjunction-representation:

\[\begin{align*}
\text{S8.1} & \implies \{\text{S8.2a}\} \\
& \quad \{\text{S8.2b}\}
\end{align*}\]

This representation is an implication-representation that contains a disjunction-representation as its consequent.

S10 If and only if
1. a given sentence is comprised of an 'or' connecting two constituent sentences of that given sentence,
then
2. we call that given sentence a disjunction-sentence,
3. that given sentence has a representation that contains a
   ' \{ \} ' connecting the representations of those con-
   stituent sentences, and
4. that representation is a disjunction-representation.

S11 A disjunction-sentence has at least two constituent sentences:
1. the constituent sentence that appears before the 'or' and
2. the constituent sentence that appears after the 'or'.
The constituent sentence that appears before the 'or' is called
the first disjunct of the disjunction-sentence; the constituent
sentence that appears after the 'or' is called the second dis-
junct. The sentence represented by 'S1.1' is a disjunction-
sentence whose first disjunct is a copy of the sentence: "A
sentence that contains two 'if-then's can be represented by a
two-arrow representation," and whose second disjunct is a
copy of the sentence: "That sentence can be represented by
a one-arrow representation."

S12 The representation of the constituent sentence that appears
before the 'or' of a given disjunction-sentence is called the
first disjunct of a disjunction-representation of that given
sentence. Thus, 'S1.1a' is the first disjunct of ' \{ S1.1a \},
and the second disjunct is 'S1.1b'.

S13 1. A sentence that contains two 'or's can sometimes be rep-
resented by a representation containing two ' \{ \} 's, or
2. a. such a sentence can sometimes be represented by a
representation containing just one ' \{ \} ', or
   b. such a sentence can always be represented by an
      elementary representation.

S14 The thirteenth statement in this section can be represented
in a variety of ways:
1. ' \{ S13.1 \} ' is one disjunction-representation of the
   thirteenth statement,
2. ' \{ \{ S13.1 \} \} ' is a second disjunction-representation of the thirteenth statement, and
3. 'S13' is an elementary representation of the thirteenth statement.

We call the first representation above a one-or disjunction-representation of the thirteenth statement; and the second one, a two-or disjunction-representation. In the two-or disjunction-representation above, the first disjunct is 'S13.1', the second disjunct is ' \{ S13.2a \} '; the first disjunct of the second disjunct is 'S13.2a', and the second disjunct of the second disjunct is 'S13.2b'.

A given disjunction-representation shows that the logical relationship of disjunction exists between the idea expressed by the disjunct that precedes the 'or' and the idea expressed by the disjunct that follows the 'or' in the disjunction-sentence which that given disjunction-representation represents.
The word in a given disjunction-sentence that denotes the logical relationship of disjunction between the ideas expressed by the disjuncts of that given disjunction-sentence is the word 'or'.

In a disjunction-sentence, the disjuncts are sentence-constituents, while the 'or' is a nonsentence-constituent. Since all of the representations of nonsentence-constituents of non-elementary representations of sentences are here called connectives, the representation of 'or' will be a connective. Like most other connectives, ' \{ \} ' is clearly distinguished from representations of sentence-constituents.

Now that we have learned how to show the logical form of implication-sentences and disjunction-sentences by means of implication-representations and disjunction-representations, it is appropriate to consider the next set of non-elementary representations, the conjunction-representations.
3. **Conjunction-Representations**

First Conjunct  |  Second Conjunct
---|---
1. a. In the last two sections we considered implication-representations and disjunction-representations,  |  b. now we are going to consider conjunction-representations.

2. a. In the last two sections we considered implication-representations and disjunction-representations,  |  b. now we are going to consider conjunction-representations.

3. S1.1a and S1.1b

4. S1.1a-S1.1b

S 2 By comparing S1.1 and S1.2, we see that 'and' is represented by '—'.

S 3 Furthermore, by comparing S1.1 and S1.3, we see that the first conjunct is represented by 'S1.1a', and the second conjunct, by 'S1.1b'.

S 4 The conjuncts of a conjunction-sentence are always sentences (as in S1.1). The conjuncts of a partial conjunction-representation may sometimes be sentences (as in S1.2), may sometimes be representations (as in S1.3), and may sometimes be combinations of both sentences and representations; but the conjuncts of a conjunction-representation will always be representations (as in S1.4).

S 5 In a conjunction-sentence, two sentences are connected by
'and' (as in S1.1). In ordinary prose, other words such as 'but', 'yet', 'still', 'while', 'moreover', and 'however', are often used to conjoin two sentences into a conjunction-sentence, but for purposes of the idealized prose being considered here, we will only discuss conjuncts that are connected by an 'and'. This same comment applies to other English words that are sometimes used in ordinary discourse to denote implication, disjunction, and the other logical relationships considered here. In a partial conjunction-representation, either
1. two sentences (as in S1.2), or combinations of sentences and representations, are connected by ' - ', or
2. two representations of sentences (as in S1.3), or combinations of sentences and representations, are connected by 'and'.

However, in a conjunction-representation, two representations of sentences are connected by ' - ' (as in S1.4).

S6 If
1. a given sentence is comprised of two constituent sentences linked by the word 'and',
then
2. that given sentence has a representation that contains a ' - ', and
3. we will say that
   a. the given sentence is a conjunction-sentence, and
   b. the idea expressed by that first constituent sentence is conjoined with the idea expressed by that second constituent sentence, i.e., the conjunction relationship exists between those ideas, and
4. we will also say that
   a. every nonelementary representation of that given sentence is a conjunction-representation, and
   b. every representation of that first constituent sentence is conjoined with every representation of that second constituent sentence.

Thus, we will say that the sentence represented by 'S1.1' is a conjunction-sentence, that the representation represented by 'S1.4' ('S1.1a-S1.1b') is a conjunction-representation, and that S1.1a is conjoined with S1.1b.

S7 If
1. a given sentence is a conjunction-sentence,
2. that given sentence has a representation that contains a ' - '.

S 8 If
1. a representation of a given sentence contains a ' - ',
then
2. a. that representation is a conjunction-representation, or
   b. that representation contains a conjunction-representation.

S 9 The following representation of the sixth statement in this section is an example of a representation that contains a conjunction-representation: \( \rightarrow \neg S6.1 \rightarrow (S6.2-S6.3-S6.4) \). The above representation is an implication-representation that contains a conjunction-representation as its consequent. The consequent is 'S6.2-S6.3-S6.4'; its first conjunct is 'S6.2', its second conjunct is 'S6.3', and its third conjunct is 'S6.4'. Note that the parentheses are used to indicate part of the syntax of the representation. If the parentheses were removed so that the representation were written as follows: \( \neg S6.1 \rightarrow S6.2-S6.3-S6.4 \), then we could not be sure that the following representation was not intended: \( (\neg S6.1 \rightarrow S6.2-S6.3) \rightarrow S6.4 \). This third representation is a conjunction-representation whose first conjunct is \( (\neg S6.1 \rightarrow S6.2-S6.3) \) and whose second conjunct is 'S6.4'. This first conjunct is an implication-representation whose antecedent is 'S6.1' and whose consequent is a conjunction-representation with 'S6.2' and 'S6.3' as its first and second conjuncts.

S 10 There is a second way of representing conjunction-sentences, which is merely a handy alternative way of representing sentences whose representations are too lengthy to fit conveniently on ordinary paper horizontally. This second way of representing conjunction-sentences is to array the conjuncts vertically and enclose them in a rectangle. Thus, the sixth statement in this section could be represented by:

\[ \rightarrow \neg S6.1 \rightarrow S6.2 \\
S6.3 \\
S6.4 \]

Similarly, the final representation in S9,

'\( (\neg S6.1 \rightarrow S6.2-S6.3) \rightarrow S6.4 \)'
would be written as follows when the conjunction-representation is written vertically:

```
>——S6.1——>S6.2-S6.3
S6.4
```

S 11 1. A sentence that contains two or more 'and's can sometimes be represented by a representation containing two or more '·'s, and
2. such a sentence can sometimes be represented by a representation containing just one '·', and
3. such a sentence can always be represented by an elementary representation.

S 12 The eleventh statement in this section is an example of a sentence that contains two or more 'and's, and it can be represented in a variety of ways:
1. 'S11.1-S11.2-S11.3' is a two-and representation of the eleventh statement, and
2. 'S11' is an elementary representation of the eleventh statement.

S 13 In order to represent the eleventh statement in this section by a one-and representation, it would be necessary to rewrite the eleventh statement as follows:
1. S11.1, and
2. a. S11.2, and
   b. S11.3.

Then, the one-and representation would be 'S13.1-S13.2', and an alternative two-and representation would be 'S13.1-(S13.2a-S13.2b)'. Note that 'S11.1-S11.2-S11.3' is a three-conjunct, two-and representation, while 'S13.1-(S13.2a-S13.2b)' is a two-conjunct, two-and representation whose second conjunct is itself a conjunction-representation, namely 'S13.2a-S13.2b'. Frequently—in fact generally—in a sentence such as the eleventh statement in this section, the 'and's between all except the last two conjuncts will be omitted. Such a sentence with the 'and's omitted is to be treated as an ellipsis for the corresponding sentence with the 'and's included. This same comment applies to 'or's.

S 14 A given conjunction-representation shows that the logical relationship of conjunction exists between the idea expressed by the conjunct that precedes the 'and' and the idea ex-
pressed by the conjunct that follows the 'and' in the conjunction-sentence which that given conjunction-representation represents. The word in a given conjunction-sentence that denotes the logical relationship of conjunction between the ideas expressed by the conjuncts of that given conjunction-sentence is the word 'and'.

S 15 In a conjunction-sentence, the conjuncts are sentence-constituents, while the 'and' is a nonsentence-constituent. Since all of the representations of nonsentence-constituents of nonelementary representations of sentences are here called connectives, the representation of 'and' will be a connective. Since ' - ' represents 'and', it is a connective. Like most other connectives, ' - ' is clearly distinguished from representations of sentence-constituents. Sentence-constituents are represented by alpha-numeric expressions, while nonsentence-constituents are usually represented by expressions other than alpha-numeric expressions.

S 16 Now that we have learned how to show the logical form of implication-sentences, disjunction-sentences, and conjunction-sentences by means of implication-representations, disjunction-representations, and conjunction-representations, it is appropriate to consider the next set of nonelementary representations, the equivalence-representations.
4. Equivalence-Representations

First Part

S1
1. a. The idea expressed by a given sentence is equivalent to the idea expressed by a second sentence

2. a. The idea expressed by a given sentence is equivalent to the idea expressed by a second sentence

b. 1. the idea expressed by that given sentence implies the idea expressed by that second sentence, and
   2. the idea expressed by that second sentence implies the idea expressed by that given sentence.

3. S1.1a if and only if S1.1b

4. S1.1a ← S1.1b ←

Second Part

S2 By comparing S1.1 and S1.2, we see that 'if and only if' is represented by '→ ← ←'.

S3 In addition to the representation of S1.1 shown in S1.4, there is also a representation of S1.1 in which the second
part is represented as a conjunction-representation, namely

\[ \text{part 1} \rightarrow \text{S1.1a} \leftarrow \text{S1.b1} \rightarrow \text{S1.b2} \rightarrow \].

S 4 The parts of an equivalence-sentence that are connected by 'if and only if' are always sentences (as in S1.1). The parts of a partial equivalence-representation may sometimes be sentences (as in S1.2), may sometimes be representations (as in S1.3), and may sometimes be combinations of both sentences and representations; but the parts of an equivalence-representation will always be representations (as in S1.4).

S 5 In an equivalence-sentence, two sentences are connected by 'if and only if' (as in S1.1), while in a partial equivalence-representation, either
1. two sentences (as in S1.2), or combinations of sentences and representations, are connected by \( \text{part 1} \rightarrow \leftarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow 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by 'S1.4' (\(\rightarrow S1.1a \leftarrow \rightarrow S1.1b \leftarrow \)) is an equivalence-representation, and that S1.1a is equivalent to S1.1b.

\[ S \]

1. We shall be able to represent a given sentence by 
\(\rightarrow Sx \leftarrow \rightarrow Sy \leftarrow \),
if and only if
2. we can represent that given sentence also by
\(\left(\rightarrow Sx \rightarrow Sy\right) \left(\rightarrow Sy \rightarrow Sx\right)\).

\[ S \]

Thus, the seventh sentence in this section can be represented by either
1. \(\rightarrow S7.1 \leftarrow \rightarrow S7.2 \leftarrow \) or
2. \(\left(\rightarrow S7.1 \rightarrow S7.2\right) \left(\rightarrow S7.2 \rightarrow S7.1\right)\).

\[ S \]

If
1. a given sentence is an equivalence-sentence,
then
2. that given sentence has a representation that contains a 
\(\rightarrow \leftrightarrow \leftarrow \leftarrow \), and
3. that given sentence has a representation that contains a pair of \(\rightarrow \leftrightarrow \leftarrow \)’s.

\[ S \]

If
1. a representation of a given sentence contains a 
\(\rightarrow \leftrightarrow \leftarrow \leftarrow \),
then
2. a. that representation is an equivalence-representation, 
or
   b. that representation contains an equivalence-representation.

\[ S \]

A given equivalence-representation shows that the logical relationship of equivalence exists between the idea expressed by the sentence that precedes the 'if and only if' and the idea expressed by the sentence that follows the 'if and only if' in the equivalence-sentence which that given equivalence-representation represents. The words in a given equivalence-sentence that denote the logical relationship of equivalence between the ideas expressed by the first and second parts of that given equivalence-sentence are the words 'if and only if'.

\[ S \]

In an equivalence-sentence, the parts connected by 'if and only if' are sentence-constituents, but the 'if and only if' is a nonsentence-constituent. Thus, \(\rightarrow \leftrightarrow \leftarrow \leftarrow \) is a connective. Like most other connectives, \(\rightarrow \leftrightarrow \leftarrow \) is clearly distinguished from representations of sentence-constituents.
Now that we have learned how to show the logical form of implication-sentences, disjunction-sentences, conjunction-sentences, and equivalence-sentences, it is appropriate to consider the next set of nonelementary representations.

5. Negation-Representations

Negated Part

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
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</thead>
<tbody>
<tr>
<td>1. It is not so that a. an elementary representation is a negation-representation.</td>
<td>Negation-sentence</td>
<td></td>
</tr>
<tr>
<td>2. N(a. an elementary representation is a negation-representation.)</td>
<td>Partial negation-representation</td>
<td></td>
</tr>
<tr>
<td>3. It is not so that S1.1a</td>
<td>Partial negation-representation</td>
<td></td>
</tr>
<tr>
<td>4. NS1.1a</td>
<td>Negation-representation</td>
<td></td>
</tr>
</tbody>
</table>

By comparing S1.1 and S1.2, we see that 'It is not so that' is represented by 'N'.

Furthermore, by comparing S1.1 and S1.3, we see that the part of the negation-sentence that is negated is represented by 'S1.1a'.

The part of a negation-sentence that follows 'It is not so that' is always a sentence (as in S1.1). The negated part of a partial negation-representation may sometimes be a sentence (as in S1.2), may sometimes be a representation (as in S1.3), and may sometimes be a combination of both sentences and representations; but the negated part of a negation-representation will always be a representation (as in S1.4).

In a negation-sentence, a sentence is preceded by 'It is not so that' (as in S1.1); while in a partial negation-representation, either
(1) a sentence (as in S1.2), or a combination of sentences and representations, is preceded by an 'N', or
(2) a representation of a sentence (as in S1.3), or a combination of sentences and representations, is preceded by 'It is not so that'.

However, in a negation-representation, a representation of a sentence is preceded by an 'N' (as in S1.4).

S 6 If
1. a given sentence is comprised of a constituent sentence preceded by a set of words 'It is not so that',
then
2. that given sentence has a representation that contains an 'N',
3. we will say that
   a. the given sentence is a negation-sentence, and
   b. the idea expressed by that given sentence is the negate of the idea expressed by that constituent sentence, i.e., that the negation relationship exists between those ideas, and
4. we will also say that
   a. every nonelementary representation of that given sentence is a negation-representation, and
   b. every representation of that given sentence is the negate of every representation of that constituent sentence.

Thus, we will say that the sentence represented by 'S1.1' is a negation-sentence, that the representation represented by 'S1.4' (i.e., 'NS1.1a') is a negation-representation, and that both 'S1.1' and 'NS1.1a' are negates of 'S1.1a'.

S 7 If
1. we wish to represent a given sentence that is comprised of a constituent sentence that is preceded and qualified by 'It is not so that',
then
2. if
   a. the representation of that constituent sentence is an implication-representation, a disjunction-representation, an equivalence-representation, a negation-representation, or an elementary representation,
   then
   b. we can represent that given sentence by writing a representation of 'It is not so that', i.e., by writing
an 'N' immediately before the representation of that constituent sentence, and

3. if
   a. the representation of that constituent sentence is a conjunction-representation,
   then
   b. we can represent that given sentence by enclosing that representation in parentheses and writing an 'N' immediately before the resulting expression.

Thus, sentences in the left column below are represented by the expressions in the right column:

| It is not so that Sx \rightarrow Sy. | N \rightarrow Sx \rightarrow Sy |
| It is not so that \{Sx\} \{Sy\} | N \{Sx\} \{Sy\} |
| It is not so that Sx \leftrightarrow Sy \leftarrow \rightarrow | N \rightarrow Sx \leftrightarrow Sy \leftarrow \rightarrow |
| It is not so that NSx. | NNSx |
| It is not so that Sx. | NSx |
| It is not so that Sx-Sy. | N(Sx-Sy) |

S 8 If
1. a representation of a given sentence contains an 'N', then
2. (a) that representation is a negation-representation, or
   (b) that representation contains a negation-representation.

S 9 A given negation-representation shows that the logical relationship of negation exists between
1. the idea expressed by the sentence which that given negation-representation represents, and
2. the idea expressed by the constituent sentence that follows the 'It is not so that' in that represented sentence.

The words in a given negation-sentence that denote the logical relationship of negation between the given negation-sentence and one of its constituent sentences are the words 'It is not so that'.

S 10 In a negation-sentence, the negated part immediately following the 'It is not so that' is a sentence-constituent, but the 'It is not so that' is a nonsentence-constituent. Thus, 'N' is
a connective, but unlike most of the other connectives, it is not as clearly distinguished from representations of sentence-constituents. The connectives other than 'N' are neither alphabetical nor numerical, whereas 'N' is only non-numerical. The sentence-constituents are both alphabetical and numerical, i.e., alpha-numeric.

S 11 After summarizing in the next section how the logical form of various kinds of sentences can be shown, we will turn to illustrating how the Internal Revenue Code might be transformed into such sentences and, when so transformed, be made easier for both human beings and automatic devices to "work with."

D. SUMMARY OF REPRESENTATIONS OF LOGICAL RELATIONS

<table>
<thead>
<tr>
<th>Logical Relationship</th>
<th>In Prose</th>
<th>In Notation</th>
</tr>
</thead>
<tbody>
<tr>
<td>implication</td>
<td>if-then</td>
<td>(\rightarrow)</td>
</tr>
<tr>
<td>disjunction</td>
<td>or</td>
<td>{ }</td>
</tr>
<tr>
<td>conjunction</td>
<td>and</td>
<td>-</td>
</tr>
<tr>
<td>equivalence</td>
<td>if and only if</td>
<td>(\leftrightarrow)</td>
</tr>
<tr>
<td>negation</td>
<td>it is not so that</td>
<td>(N)</td>
</tr>
</tbody>
</table>

II. SAMPLES OF THE GROSS ESTATE PROVISIONS OF THE INTERNAL REVENUE CODE IN THE FORM OF STRUCTURED STATEMENTS

A. INTRODUCTION

Transforming a given section of the Internal Revenue Code into the more logically organized form discussed in Part I of this Article will be illustrated by section 2031, which defines "gross estate" for purposes of the estate tax. The process of transforming a given section is still somewhat more artful than systematic; the resulting restatement of a given section is likely to vary in some particulars from analyst to analyst. The broad strategy is to transform a given section by breaking it down into conveniently small constituent statements, and then recombining those constituent statements in a way that will (1) make explicit the logical relationships (as understood by the analyst) between those statements, and at the
same time, (2) assert economically what the section is intended to include.

In the course of transforming section 2031 into the form of a "structured statement" (structured so that the logical relationships intended are made explicit), it is interesting to consider questions about what tasks are most appropriately done by human analysts and what tasks are sufficiently routine that they are best handled by machines. The boundary line of what is appropriate here is only hazily defined (if at present defined at all) and subject to change with changes in the state of our knowledge. In a field that is probably still in its beginning stages, it is perhaps only the incautious who venture to assert dogmatically either what is possible or what is appropriate. The evaluations offered here are tendered most tentatively.

Consider the following situation. Sam Jones, a native New Haven resident and part-owner of a boarding house for tourists in the Virgin Islands, made the long journey of no return when his car skidded out of control on an icy road. One of the problems facing Sam's executor in filing the estate tax return is deciding what effect section 2031 has upon Sam's interest in the boarding house for purposes of the federal tax on Sam's estate. Section 2031 of the Internal Revenue Code of 1954 defines gross estate as follows:

(a) GENERAL.—The value of the gross estate of the decedent shall be determined by including to the extent provided for in this part, the value at the time of his death of all property, real or personal, tangible or intangible, wherever situated, except real property situated outside of the United States.

(b) VALUATION OF UNLISTED STOCK AND SECURITIES.—In the case of stock and securities of a corporation the value of which, by reason of their not being listed on an exchange and by reason of the absence of sales thereof, cannot be determined with reference to bid and asked prices or with reference to sales prices, the value thereof shall be determined by taking into consideration, in addition to all other factors, the value of stock or securities of corporations engaged in the same or a similar line of business which are listed on an exchange.

The executor might well marshal the relevant facts in the form of the following set of four "factual statements":

FS 1 Sam Jones is the decedent.

FS 2 At the time of his death, Sam Jones had a one-third interest in some property.

FS 3 That property is a boarding house.
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FS 4 That boarding house is located in the Virgin Islands.

In order to highlight some of the decisions that the executor will be called upon to make, it will be helpful to look upon section 2031 as asserting that when certain specified legal conditions are fulfilled, certain specified legal consequences will follow. The relevant parts of section 2031 for this example, namely those parts in section 2031(a), might be summarized by the following set of statements:

S 1 A decedent had an interest in property at the time of his death.
S 2 That property is tangible property.
S 3 That property is real property.
S 4 That property is situated in the United States.
S 50 The value of the gross estate of that decedent shall be determined by including, to the extent provided for in part III, the value of that property at the time of his death.

In the examples to be considered here, conditions will be expressed by statements 'S0' through 'S38' and consequences will be expressed by statements 'S50' through 'S65'.

For purposes of this article, a statement which asserts that if certain specified legal conditions are fulfilled, then certain legal consequences will follow, will be called a "norm." The norms expressed by section 2031(a) may be represented by the following five representations:

R 1 \[ \text{A decedent had an interest in property at the time of his death} \rightarrow \text{that property is tangible property} \rightarrow \text{that property is personal property} \rightarrow \text{the value of the gross estate of that decedent shall be determined by including to the extent provided for in part III, the value of that property at the time of his death.} \]
It should be noted that a norm is represented just like any other statement. The five representations of norms above indicate that there are four sets of conditions that, when fulfilled, lead to consequence-S50 and one set of conditions that, when fulfilled, leads to consequence-NS50.

But how shall it be decided whether the states of affairs described by statements FS1 through FS4 fulfill any of the conditions expressed by statements S1 through S4? Is this decision a task for a machine? Or is it a task for a lawyer? In this writer's opinion, this is the crucial task that should continue to be done by a human brain rather than an electronic or mechanical "brain." It is the lawyer, and not a machine, that should characterize the events that have occurred and the states of affairs that prevail, in terms of the language of the norms of the community. It is he that should make the decisions about whether or not given events or states of affairs fulfill specified sets of conditions. By varying his characterizations as changes in the context warrant it, a human decision-maker can build into the legal system a sensitivity and flexibility that is desirable in order to keep the legal system in harmony with a society that may be rapidly changing in many respects. It is somewhat difficult to envision at this time how an electronic device could ever be programmed to achieve the same degree of sensitivity or flexibility.

But what are the decisions that need to be made in this example? One question is whether the decedent had an interest in property at the time of his death, that is, whether FS1 and FS2 fulfill the condition expressed by S1. The answer to this question is clearly in the affirmative. Following the general style previously employed in representing statements, questions and answers can be represented as follows:

1. a '(?)' will be used in representing a question,
2. a '(1)' will be used in representing an affirmative answer, and
3. a '(0)' will be used in representing a negative answer.
The above question will be represented by:

\[ Q_1 \rightarrow \text{FS1-FS2-}(?)\rightarrow S1 \]

An answer of "Yes" to the question whether FS1 and FS2 fulfill S1, will be represented by:

\[ A1.1 \rightarrow \text{FS1-FS2-}(1)\rightarrow S1 \]

An answer of "No" to the same question will be represented by:

\[ A1.0 \rightarrow \text{FS1-FS2-}(0)\rightarrow S1 \]

Some of the other questions that the executor of Sam's estate will have to decide before he can come to some conclusion about how to complete the estate tax return are the following:

\[ Q_2 \rightarrow \text{FS3-}(?)\rightarrow S2 \]
\[ Q_3 \rightarrow \text{FS3-}(?)\rightarrow \text{NS2} \]
\[ Q_4 \rightarrow \text{FS3-}(?)\rightarrow S3 \]
\[ Q_5 \rightarrow \text{FS3-}(?)\rightarrow \text{NS3} \]
\[ Q_6 \rightarrow \text{FS4-}(?)\rightarrow S4 \]
\[ Q_7 \rightarrow \text{FS4-}(?)\rightarrow \text{NS4} \]

After the characterizations have been made by a lawyer with respect to which conditions have and have not been fulfilled, there is still the problem of ascertaining what legal consequences should follow from those fulfilled conditions. Once again we encounter a problem that may or may not be suitable for automation. If the Internal Revenue Code were transformed into the form of the "structured statements" specified in Part I of this Article, then the task of ascertaining their legal consequences is routine enough that it could be done by means of some automatic device. When it is economically feasible to do so, the decisions involved in matching consequences to fulfilled conditions probably ought to be done automatically. However, to the extent that lawyers continue to perform this task, it may be worthwhile making the task simpler by having the Internal Revenue Code available in the form of "structured statements." The analysis by lawyers would be facilitated if the five norms represented by R1 through R5 were consolidated into one or more complex norms that say the same things as the five norms represented above.

There is an advantage in working with representations when the analyst is seeking to ascertain the most economical way of combining the statements expressing conditions and consequences to
express the entire set of norms asserted in a given section of the Internal Revenue Code. When dealing with representations rather than prose statements, it is easier to see how various constituent norms can be consolidated into a complex norm so that all of the various constituent norms are asserted by that complex norm. For example, it is evident that R1 and R2 have common constituents S1, NS3, and S50, and can be consolidated into a complex norm that can be represented by:

\[ R_{1-2} \rightarrow S_1 - \{ S_2 \} - NS_3 \rightarrow S_{50} \]

Similarly, it is evident that R3 and R4 have common constituents S1, S4, and S50, and can be consolidated into a complex norm that can be represented by:

\[ R_{3-4} \rightarrow S_1 - \{ S_2 \} - S_4 \rightarrow S_{50} \]

In turn, it becomes evident that R1-2 and R3-4 have common constituents S1, \{ S_2 \}, and S50, and can be consolidated into a complex norm that can be represented by:

\[ R_{1-4} \rightarrow S_1 - \{ S_2 \} - (NS_3) \rightarrow S_{50} \]

We know that 'If A, then if B, then C' is just another way of saying 'If A and B, then C'. Similarly, the norm represented by R1-4 says the same thing as the norm represented by:

\[ R_{1-4a} \rightarrow S_1 \rightarrow (NS_3) \rightarrow S_{50} \]

And similarly, the norm represented by R5 says the same thing as the norm represented by:

\[ R_{5a} \rightarrow S_{1} \rightarrow S_{3-ns_{4}} \rightarrow ns_{50} \]

We know that 'not-(not-A or B)' is just another way of saying 'A and not-B'. Similarly, the antecedent of the consequent of R5a can be expressed as 'N \{ NS_3 \}' in place of 'S3-NS4', and thus, the norm
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represented by R5a can be seen to say the same thing as the norm represented by:

\[ \text{R5b} \rightarrow S1 \rightarrow N \begin{cases} \{S2\} \\ \{NS2\} \end{cases} \rightarrow NS50 \]

We know that 'A and (B or not-B)' says the same thing as 'A'.

Similarly, '{S2}' can be added to the antecedent of R5b as a conjunct without changing what is said. Thus, the norm represented by R5b says the same thing as the norm represented by:

\[ \text{R5c} \rightarrow S1 \begin{cases} \{S2\} \\ \{NS2\} \end{cases} \rightarrow N \begin{cases} \{NS3\} \\ \{S4\} \end{cases} \rightarrow NS50 \]

When R5c and R1-4a are compared, they are seen to have similar antecedents, and it is evident (because '\(\rightarrow NSx \rightarrow NSy\)' says the same thing as '\(\rightarrow Sy \rightarrow Sx\)') that their consequents may be consolidated to form an equivalence. Thus, R5c and R1-4a may be consolidated into a complex norm that can be represented by:

\[ \text{R1-5} \rightarrow S1 \begin{cases} \{S2\} \\ \{NS2\} \end{cases} \rightarrow N \begin{cases} \{NS3\} \\ \{S4\} \end{cases} \leftrightarrow \text{S50} \]

Only one more modification is necessary before we have the final representation of a structured statement version of the present section 2031(a). There is always at least one, and there are frequently more, essential conditions that are implicit rather than express in the sections of the Internal Revenue Code as currently drafted. For example, there is the following implicit condition in every section: "This section is not unconstitutional." To expressly provide for such implicit conditions, we shall introduce a residuary condition that will be included in every section. The statement expressing this residuary condition will be represented by an 'SO', and it is stated as follows: "All other essential conditions are fulfilled." When the residuary condition is added to the norm represented by R1-5, the resulting norm is the one represented by:

\[ \text{R1-5a} \rightarrow SO \rightarrow S1 \begin{cases} \{S2\} \\ \{NS2\} \end{cases} \rightarrow \begin{cases} \{NS3\} \\ \{S4\} \end{cases} \leftrightarrow \text{S50} \]

Now that a structured statement version of section 2031(a) has been completely represented, the next thing to consider is the representation of a structured statement version of section 2031(b). One possible breakdown of the present version of section 2031(b)
is into the set of conditions and consequences expressed by the following statements:

\[ \text{S 5} \quad \text{That property is stock or securities of a corporation.} \]

\[ \text{S 6} \quad \text{The value of that stock or those securities, by reason of their not being listed on an exchange, cannot be determined with reference to bid and asked prices.} \]

\[ \text{S 7} \quad \text{The value of that stock or those securities, by reason of the absence of sales thereof, cannot be determined with reference to sales prices.} \]

\[ \text{S 51} \quad \text{The value of that property shall be determined by taking into consideration, in addition to all other factors, the value of stock or securities of corporations} \]

1. engaged in the same or a similar line of business
2. that are listed on an exchange.

One way of representing a structured statement of the norm expressed by section 2031(b) is by:

\[ \text{R 6} \quad \rightarrow \text{S1-S5-S6-S7} \rightarrow \text{S51} \]

The norm represented by R6, in turn, says the same thing as the norm represented by:

\[ \text{R 6a} \quad \rightarrow \text{S1} \rightarrow \rightarrow \text{S5-S6-S7} \rightarrow \text{S51} \]

We know that a ‘{S2}\{NS2\}’ can be conjoined to the antecedent of R6a without changing what is represented. Thus, the norm represented by R6a says the same thing as the norm represented by:

\[ \text{R 6b} \quad \rightarrow \text{S1}\{S2\}\{NS2\} \rightarrow \rightarrow \text{S5-S6-S7} \rightarrow \text{S51} \]

Finally, the residuary condition is added to obtain a final representation of a structured statement version of section 2031(b) that can conveniently be consolidated with the R1-5a representation of the structured statement version of section 2031(a). This final structured statement version of 2031(b) can be represented by:

\[ \text{R 6c} \quad \rightarrow \text{S0-S1}\{S2\}\{NS2\} \rightarrow \rightarrow \text{S5-S6-S7} \rightarrow \text{S51} \]

Comparison of R1-5a and R6c indicates that they both have the
same antecedent, \( \text{'-SO-S1} - \{ S_2 \} \), and therefore, may be consolidated into:

\[
R_1 - 6 \rightarrow \text{'SO-S1} - \{ S_2 \}_{NS_2} \rightarrow \text{NS}_3 \rightarrow S_4 \rightarrow S_5 - S_6 - S_7 \rightarrow S_{51}
\]

The description in prose of the transformation of the six norms represented by \( R_1 \) through \( R_6 \) into the single complex norm represented by \( R_{1-6} \) may be somewhat misleading to some readers. The process is not really as complex as the description may lead one to believe. In fact, this transformation process is sufficiently routine that it could be done automatically by machine.

The prose version represented by \( R_{1-6} \), which is what is here called a “structured statement,” is shown in the next section of this article as SS-2031. Immediately preceding SS-2031, there is a diagram labeled ‘AD-2031’; there is a similarly labeled diagram preceding the structured statement of section 2041 as well. These “arrow diagrams” are representations of the structured statements that follow them. The arrow diagrams are included to emphasize the intended syntax of the structured statements. The arrow diagrams, however, are superfluous, and with a little practice, a reader can learn to discern the syntax just as readily from the structured statements as he can from the arrow diagrams.

A word of caution should be added with respect to the structured statements of sections 2031 and 2041 that are included here. These structured statements have been prepared on the basis of the text of the sections alone without consulting the regulations, cases, or other materials that would shed light upon the appropriate interpretations that either have been given or ought to be given to the sections considered. The versions presented here undoubtedly contain ambiguities, errors of judgment, and commitments to one of various alternative interpretations of a given section (about which the writer would probably change his mind after further research), as well as some omissions and outright clerical mistakes. But even though the substance of the structured statements for these particular sections may be imperfect, the essential point about the possibility of a more organized drafting of the Internal Revenue Code should nevertheless be clear. To the extent that tax materials
are more systematically organized in such a fashion, both lawyers and machines will be able to work with them more effectively.

There is no reason why drafting in the form of structured statements should be confined to the statutory tax materials. In fact, it would seem that the Treasury Regulations are more appropriate materials with which to begin such systematic drafting because rewriting the regulations does not involve all the complications of the legislative action that would be required to modify the Code.

B. Section 2031: Definition of Gross Estate

AD-2031 > SO-S1-\{S2\} \rightarrow \{S3\} \leftarrow S50 \leftarrow S51

SS-2031

If

1. All other essential conditions are fulfilled, SO
2. a decedent had an interest in property at the time of his death, and S1
3. a) that property is tangible property, or S2
   b) that property is not tangible property, NS2
then

4. if and only if
   a. 1) that property is personal property, or NS3
      2) that property is situated in the United States, S4
   then
      b. the value of the gross estate of that decedent shall be determined by including to the extent provided for in part III, the value of that property at the time of his death, and S50

5. if
   a. that property is stock or securities of a corporation, S5
   b. the value of that stock or those securities, by reason of their not being listed on an exchange, cannot be determined with reference to bid and asked prices, and S6
   c. the value of that stock or those securities, by reason of the absence of sales thereof, cannot be determined with reference to sales prices, S7
then
d. the value of that property shall be determined by taking into consideration, in addition to all other factors, the value of stock or securities of corporations:

1. engaged in the same or a similar line of business
2. that are listed on an exchange.

C. SECTION 2041: POWERS OF APPOINTMENT

The advantages of a more systematically drafted statute in making the lawyer's task easier in working with that statute can be somewhat better appreciated when dealing with sections of the Code that are relatively complex. A structured statement version of the moderately complex section dealing with powers of appointment is presented below as illustrative. After struggling through the present version of section 2041, the reader is left to his own judgment as to how much the communication of the draftsman's message is improved when the syntactical connections between the various constituent parts is made more explicit in AD-2041 and SS-2041.

Section 2041 of the Internal Revenue Code of 1954 deals with powers of appointment as follows:

(a) IN GENERAL.—The value of the gross estate shall include the value of all property (except real property situated outside of the United States)—

(1) POWERS OF APPOINTMENT CREATED ON OR BEFORE OCTOBER 21, 1942.—To the extent of any property with respect to which a general power of appointment created on or before October 21, 1942, is exercised by the decedent—

(A) by will, or

(B) by a disposition which is of such nature that if it were a transfer of property owned by the decedent, such property would be includible in the decedent's gross estate under sections 2035 to 2038, inclusive:

but the failure to exercise such a power or the complete release of such a power shall not be deemed an exercise thereof. If a general power of appointment created on or before October 21, 1942, has been partially released so that it is no longer a general power of appointment, the exercise of such power shall not be deemed to be the exercise of a general power of appointment if—

(i) such partial release occurred before November 1, 1951, or

(ii) the donee of such power was under a legal disability to release such power on October 21, 1942, and
such partial release occurred not later than 6 months after the termination of such legal disability.

(2) Powers Created After October 21, 1942.—To the extent of any property with respect to which the decedent has at the time of his death a general power of appointment created after October 21, 1942, or with respect to which the decedent has at any time exercised or released such a power of appointment by a disposition which is of such nature that if it were a transfer of property owned by the decedent, such property would be includible in the decedent's gross estate under sections 2035 to 2038, inclusive. A disclaimer or renunciation of such a power of appointment shall not be deemed a release of such power. For purposes of this paragraph (2), the power of appointment shall be considered to exist on the date of the decedent's death even though the exercise of the power is subject to a precedent giving of notice or even though the exercise of the power takes effect only on the expiration of a stated period after its exercise, whether or not on or before the date of the decedent's death notice has been given or the power has been exercised.

(3) Creation of Another Power in Certain Cases.—To the extent of any property with respect to which the decedent—

(A) by will, or

(B) by a disposition which is of such nature that if it were a transfer of property owned by the decedent such property would be includible in the decedent's gross estate under section 2035, 2036, or 2037,

exercises a power of appointment created after October 21, 1942, by creating another power of appointment which under the applicable local law can be validly exercised so as to postpone the vesting of any estate or interest in such property, or suspend the absolute ownership or power of alienation of such property, for a period ascertainable without regard to the date of the creation of the first power.

(b) Definitions.—For purposes of subsection (a)—

(1) General Power of Appointment.—The term "general power of appointment" means a power which is exercisable in favor of the decedent, his estate, his creditors, or the creditors of his estate; except that—

(A) A power to consume, invade, or appropriate property for the benefit of the decedent which is limited by an ascertainable standard relating to the health, education, support, or maintenance of the decedent shall not be deemed a general power of appointment.

(B) A power of appointment created on or before October 21, 1942, which is exercisable by the decedent only in conjunction with another person shall not be deemed a general power of appointment.

(C) In the case of a power of appointment created after
October 21, 1942, which is exercisable by the decedent only in conjunction with another person—

(i) If the power is not exercisable by the decedent except in conjunction with the creator of the power — such power shall not be deemed a general power of appointment.

(ii) If the power is not exercisable by the decedent except in conjunction with a person having a substantial interest in the property, subject to the power, which is adverse to exercise of the power in favor of the decedent — such power shall not be deemed a general power of appointment. For the purposes of this clause a person who, after the death of the decedent, may be possessed of a power of appointment (with respect to the property subject to the decedent's power) which he may exercise in his own favor shall be deemed as having an interest in the property and such interest shall be deemed adverse to such exercise of the decedent's power.

(iii) If (after the application of clauses (i) and (ii)) the power is a general power of appointment and is exercisable in favor of such other person — such power shall be deemed a general power of appointment only in respect of a fractional part of the property subject to such power, such part to be determined by dividing the value of such property by the number of such persons (including the decedent) in favor of whom such power is exercisable.

For purposes of clauses (ii) and (iii), a power shall be deemed to be exercisable in favor of a person if it is exercisable in favor of such person, his estate, his creditors, or the creditors of his estate.

(2) LAPSE OF POWER.—The lapse of a power of appointment created after October 21, 1942, during the life of the individual possessing the power shall be considered a release of such power. The preceding sentence shall apply with respect to the lapse of powers during any calendar year only to the extent that the property, which could have been appointed by exercise of such lapsed powers, exceeded in value, at the time of such lapse, the greater of the following amounts:

(A) $5,000, or

(B) 5 percent of the aggregate value, at the time of such lapse, of the assets out of which, or the proceeds of which, the exercise of the lapsed powers could have been satisfied.

(3) DATE OF CREATION OF POWER.—For purposes of this section, a power of appointment created by a will executed on or before October 21, 1942, shall be considered a power created on or before such date if the person executing such will dies before July 1, 1949, without having republished such will, by codicil or otherwise, after October 21, 1942.
If

1. All other essential conditions are fulfilled, SO
2. a) 1. a general power of appointment with respect to given property has been created on or before October 21, 1942 (see subsection 15c), and S8
   2. that general power of appointment has been exercised by the decedent (see subsections 5b, 6b, and 7d)
      A) by will, or
      B) by a disposition that is of such nature that if it were a transfer of property owned by the decedent, that property would be includible in the decedent’s gross estate under section 2035 to 2038, inclusive, S9
b) 1. a general power of appointment with respect to given property has been created after October 21, 1942 (see subsection 15c), and S10
   2. A) the decedent had that power of appointment at the time of his death (see subsection 9), or S11
      B) the decedent has at some time exercised or released (see subsections 8b and 14b) that power of appointment by a disposition that is of such nature that if it were a transfer of property owned by the decedent, that property would be includible in the decedent’s gross estate under sections 2035 to 2038, inclusive, or S12
c) 1. a power of appointment with respect to given property has been created after October 21, 1942 (see subsection 15c), S13
   2. that power of appointment has been exercised by the decedent
      A) by will, or
      B) by a disposition that is of such nature that if it were a transfer of property owned by the decedent, that property would be includible in the decedent’s gross estate under section 2035, 2036, or 2037, and S14
3. that power of appointment has been exercised
by creating another power of appointment that under the applicable local law
A) can be validly exercised so as to postpone the vesting of any estate or interest in such property, or
B) suspend the absolute ownership or power of alienation of such property,
for a period ascertainable without regard to the date of creation of the first power, and

3. a) that property is not real property, or
b) that property is situated in the United States,
then

4. the value of the gross estate shall include the value of that property,

5. if
   a. there has been a failure to exercise that given general power of appointment,
then
   b. for purposes of subsection 2a2, that failure shall not be deemed to be an exercise of that power,

6. if
   a. that general power of appointment has been completely released (see subsections 8b and 14b),
then
   b. for purposes of subsection 2a2, that complete release shall not be deemed to be an exercise of that power,

7. if
   a. that general power of appointment has been partially released (see subsections 8b and 14b) so that it is no longer a general power of appointment, and
   b. 1) that partial release occurred before November 1, 1951, or
      2) A. the donee of that power was under a legal disability to release that power on October 21, 1942, and
         B. that partial release occurred not later than six months after the termination of that legal disability, and
   c. that power is exercised,
   then
   d. for purposes of subsection 2a2, that exercise of
that power shall not be deemed to be the exercise of a general power of appointment,

8. if
   a. there is a disclaimer or renunciation of that power of appointment,
   then
   b. for purposes of subsection 2b2B, that disclaimer or renunciation shall not be deemed to be a release of that power,

9. for purposes of subsection 2b2A, the power shall be considered to exist on the date of decedent's death
   a. 1) even though the exercise of the power is subject to a precedent giving of notice, or
      2) even though the exercise of the power takes effect only on the expiration of a stated period after its exercise,
   b. whether or not on or before the decedent's death
      1) notice has been given, or
      2) the power has been exercised,

10. if
   a. 1) A. that power is to consume, invade, or appropriate property, for the benefit of the decedent, and
      B. that power is limited by an ascertainable standard relating to the health, education, support, or maintenance, of the decedent, or
   2) A. that power is exercisable by the decedent only in conjunction with another person, and
      B. 1) that power was created on or before October 21, 1942 (see subsection 15c), or
      2) a. that power was created after October 21, 1942 (see subsection 15c), and
      b. 1) that power is not exercisable by the decedent except in conjunction with the creator of that power, or
      2) that power is not exercisable
by the decedent except in con-
junction with a person having
a substantial adverse interest
in the property (see subsection
10c) subject to the power,
which is adverse to the exer-
cise of the power, S30

then
b. for purposes of section 2041, that power shall
not be deemed a general power of appointment,
and S58
c. for purposes of subsection 10a2B-2b2, a person
who, after the death of the decedent, may be
possessed of a power of appointment (with re-
spect to the property subject to the decedent's
power) that he may exercise in his own favor
(see subsection 12b) shall be deemed as having
an interest adverse to such exercise of the de-
cedent's power, S59

11. if
a. that power was created after October 21, 1942
(see subsection 15c), S28
b. that power is exercisable by the decedent only
in conjunction with another person, S27
c. after the application of subsections 10a2B-2b1
and 10a2B-2b2, that power is a general power
of appointment, and S31
d. that power is exercisable in favor of that other
person (see subsection 12b), S32
then

e. for purposes of section 2041, that power shall
be deemed a general power of appointment
only in respect of a fractional part of the prop-
erty subject to that power, and S60
f. that fractional part shall be determined by
dividing the value of that property by the
number of such persons (including the dece-
dent) in favor of whom such power is exercis-
able (see subsection 12b), S61

12. if
a. a power is exercisable in favor of
1) a person,
2) his estate,
3) his creditors, or
4) the creditors of his estate,
then
b. for purposes of subsections 10c, 11d, and 11f, that power shall be deemed to be exercisable in favor of that person,

13. if
a. a power of appointment is exercisable in favor of
   1) a person,
   2) his estate,
   3) his creditors, or
   4) the creditors of his estate, and
b. it is not otherwise provided in section 2041 that such a power shall not be deemed a general power of appointment,
then
  c. that power shall be deemed a general power of appointment,

14. if
a. a power of appointment lapses during the life of the individual possessing that power,
then
b. for purposes of section 2041, that lapse of that power during any calendar year shall be considered a release of that power to the extent that the property, which could have been appointed by exercise of that lapsed power, exceeded in value, at the time of the lapse, the greater of the following amounts:
   1) $5,000, or
   2) 5 percent of the aggregate value, at the time of that lapse, of the assets out of
      A) which, or
      B) the proceeds of which,
      the exercise of the lapsed powers could have been satisfied, and

15. if
a. a power of appointment is created by a will executed on or before October 21, 1942, and
b. the person executing that will died before July 1, 1949, without having republished that will, by codicil or otherwise, after October 21, 1942,
c. for purposes of section 2041, a power of appointment created by that will shall be considered a power created on or before October 21, 1942.

If the structured statement version of section 2041 seems complicated, it may be because it says so much. A careful perusal will reveal that no less than 168 implications are asserted by SS-2041. Each of the eight items in column A can be combined with each of the twenty-one items in column B to form a representation of one of the implications asserted by SS-2041.

<table>
<thead>
<tr>
<th>Column A</th>
<th>Column B</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. &gt;--SO-S8-S9-NS3--&gt;</td>
<td>1. S52</td>
</tr>
<tr>
<td>2. &gt;--SO-S8-S9-S4-- &gt;--S16--S53</td>
<td></td>
</tr>
<tr>
<td>3. &gt;--SO-S10-S11-NS3--&gt;</td>
<td>3. &gt;--S17--S54</td>
</tr>
<tr>
<td>4. &gt;--SO-S10-S11-S4-- &gt;--S18-S19-S22--S55</td>
<td></td>
</tr>
<tr>
<td>5. &gt;--SO-S10-S12-NS3-- &gt;--S18-S20-S21-S22--S55</td>
<td></td>
</tr>
<tr>
<td>6. &gt;--SO-S10-S12-S4-- &gt;--S23--S56</td>
<td></td>
</tr>
<tr>
<td>8. &gt;--SO-S13-S14-S15-S4-- &gt;--S27--S59</td>
<td></td>
</tr>
<tr>
<td>9. &gt;--S26-S28-S29--S59</td>
<td></td>
</tr>
<tr>
<td>10. &gt;--S26-S27--S58</td>
<td></td>
</tr>
<tr>
<td>11. &gt;--S26-S27--S59</td>
<td></td>
</tr>
<tr>
<td>12. &gt;--S26-S28-S29--S58</td>
<td></td>
</tr>
<tr>
<td>13. &gt;--S26-S28-S29--S59</td>
<td></td>
</tr>
<tr>
<td>14. &gt;--S26-S28-S30--S58</td>
<td></td>
</tr>
<tr>
<td>15. &gt;--S26-S28-S30--S59</td>
<td></td>
</tr>
<tr>
<td>16. &gt;--S28-S27-S31-S32--S60</td>
<td></td>
</tr>
<tr>
<td>17. &gt;--S28-S27-S31-S32--S61</td>
<td></td>
</tr>
<tr>
<td>18. &gt;--S33--S62</td>
<td></td>
</tr>
<tr>
<td>19. &gt;--S34-S35--S63</td>
<td></td>
</tr>
<tr>
<td>20. &gt;--S36--S64</td>
<td></td>
</tr>
<tr>
<td>21. &gt;--S37-S38--S65</td>
<td></td>
</tr>
</tbody>
</table>

For example, the representation of A2-B6 is

' >--SO-S8-S9-S4-- >--S23--S56'.

III. AUTOMATIC INFORMATION RETRIEVAL

The problem of finding material in a statute is essentially a problem of communication—not of law—although inevitably finding absorbs much of a lawyer's time. Interpreting, predicting, planning—these are the elements of the legal decision making process with which the lawyer is especially trained to deal. It is the task of the machine to present him, in usable form, with all the raw material on which to base his decision.¹

¹ Helgeson, A Preliminary Design for Coding Statutes to Punched Cards, 1957 (unpublished paper at Yale Law School).
There was described in Part I a more systematic form into which written tax materials might be transformed. In Part II, a sample part of the Internal Revenue Code, sections 2031 and 2041, was transformed into "structured statement" form. It remains to be shown in this part some of the kinds of operations that mechanical and electronic devices will be able to perform on tax materials that are in the form of structured statements.

A structured statement of a given section is a separation of that section into parts that themselves are sentences with an explicit specification of the logical relationships between those sentences. Since this is the case, it seems reasonable to consider to what extent the kind of reasoning that human analysts do with respect to sentences and the logical relationships between the ideas expressed by those sentences can be duplicated by means of automatic devices.

Consider the following pair of statements:

\[ S_1 \]
If
1. some of the work of lawyers can be done by machines,
then
2. lawyers will have more time to spend on important problems.

\[ S_2 \]
Some of the work of lawyers can be done by machines.

If S1 and S2 were given as premises, what conclusion would a human analyst deduce? Most rational human analysts would agree that S1.2 is a reasonable conclusion, given S1 and S2 as premises. Could an automatic device be used to deduce the same conclusion? For purposes of mechanical deduction by means of edge-punched cards, we shall deal with representations of the sentence-patterns of which the sentences represented by 'S1.1', 'S1.2', and 'S2' are occurrences. Sentence-patterns will be represented by alpha-numeric expressions that begin with a capital 'P'. The sentence-pattern of which the antecedent of the first premise above is an occurrence will be represented by a 'P1'. The sentence-pattern of which the consequent of the first premise is an occurrence will be represented by a 'P4'. Because the second premise is just a second occurrence, the same sentence-pattern that is stated by the antecedent of the first premise, the sentence-pattern of which the second premise is an occurrence, will also be represented by a 'P1'. Similarly, because the conclusion is just a second occurrence of the same sentence-pattern that is stated by the consequent of the second premise, the sentence-pattern of which the conclusion is an occurrence will
also be represented by a 'P4'. With these representations, the argument might be partially represented as follows:

from
1. \( \text{If } P1 \text{ then } P4 \)
and
2. \( P1 \)
it is valid to infer
3. \( P4 \).

Notice that deducing 'P4' from 'If P1 then P4' and 'P1' seems reasonable no matter what statement it is that 'P1' or 'P4' represents. One mechanical device that can perform the same task that a human analyst does in reasoning as above is a set of edge-punched cards. Of course, everything that can be done by means of edge-punched cards can also be done by means of more sophisticated data processing systems employing center-punched cards, magnetic tape, or other means of storing information in machine-readable form. The operations that can be performed mechanically will be described here in terms of edge-punched cards because in this way the operations can probably be more easily visualized.

Since the corpus of information in this example contains only two elements—one condition, expressed by an occurrence of P1, and one consequence, expressed by an occurrence of P4—we would need only four (two squared) cards to represent this corpus. However, we shall want to consider more complex examples that have as many as four elements. Therefore, a set of sixteen (four squared) cards will be used for even this first example.

Each card will have four holes in it, the first hole for P1, the second hole for P2, the third hole for P3, and the fourth hole for P4. Each hole will be either a 'U' or an 'O'. A 'U' for hole-P1 will indicate that the condition expressed by an occurrence of P1 is fulfilled, whereas an 'O' for hole-P1 will indicate that the condition expressed by an occurrence of P1 is not fulfilled. Similarly, a 'U' and an 'O' for hole-P2 and hole-P3 will indicate the fulfillment and nonfulfillment of the conditions expressed by occurrences of P2 and P3. A 'U' for hole-P4 will indicate that the consequence expressed by an occurrence of P4 does follow, whereas an 'O' for
hole-P4 will indicate that the consequence expressed by an occurrence of P4 does not follow. The sixteen cards will look like this:

<table>
<thead>
<tr>
<th>Card</th>
<th>P1</th>
<th>P2</th>
<th>P3</th>
<th>P4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Card 0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
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<td>U</td>
</tr>
<tr>
<td>Card 2</td>
<td>0</td>
<td>0</td>
<td>U</td>
<td>0</td>
</tr>
<tr>
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<td>0</td>
</tr>
<tr>
<td>Card 4</td>
<td>0</td>
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<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Card 5</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>U</td>
</tr>
<tr>
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</tr>
<tr>
<td>Card 7</td>
<td>0</td>
<td>U</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Card 8</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Card 9</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>U</td>
</tr>
<tr>
<td>Card 10</td>
<td>0</td>
<td>0</td>
<td>U</td>
<td>0</td>
</tr>
<tr>
<td>Card 11</td>
<td>0</td>
<td>U</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Card 12</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Card 13</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>U</td>
</tr>
<tr>
<td>Card 14</td>
<td>0</td>
<td>0</td>
<td>U</td>
<td>0</td>
</tr>
<tr>
<td>Card 15</td>
<td>0</td>
<td>U</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

The problem has just two premises. The first premise ('If P1, then P4') indicates that the combination of P1 and NP4 is not permitted, and therefore, all cards bearing this combination must be eliminated. To eliminate this combination, the sixteen cards are stacked in a deck and a needle is inserted through hole-P1. When the needle is lifted, cards 0, 1, 2, 3, 4, 5, 6, and 7 hang on the needle, while the others drop out. The drop-outs are all P1 cards, while the hang-ons are all NP1 cards. The P1 drop-outs are then stacked and needled through hole-P4. Cards 8, 10, 12, and 14, which are NP4 cards, will hang on, while cards 9, 11, 13, and 15, which are P4 cards, will drop out. The set of cards now hanging on the needle is the set of all cards in which the combination of P1 and NP4 occur. This is the set that is to be excluded. Therefore, the set comprised of the other twelve cards (0–7, 9, 11, 13, and 15) is the set of cards that is compatible with the first premise.

The next step is to select from the set of cards compatible with
the first premise, a second set of cards that is compatible with the second premise. The second premise is P1, so we merely stack the set compatible with the first premise and needle it through hole-P1. The drop-outs are all P1 cards, and they are the set of cards that is compatible with both premises. By stacking these drop-outs and inspecting the set, which will consist of cards 9, 11, 13, and 15, it will be seen that every card of the set has a 'U' in hole-P1 and a 'U' in hole-P4, and that there are no other holes in which every card has a 'U'. The set will look as follows:

```
  U 0 0 U
P1 P2 P3 P4
```

Set of Cards
9, 11, 13, 15

The element represented by a given hole will be a valid conclusion from a set of specified premises if and only if every card in the set of cards that is compatible with every one of those specified premises has a 'U' in that given hole. Thus, ideas expressed by occurrences of P1 and P4 are both valid conclusions from the specified pair of premises. Thus, by the specified mechanical operations on a set of edge-punched cards, it can be determined that consequence-P4 does follow from the pair of premises, P1 and $\neg P1 \rightarrow P4$.

The operation of the edge-punched cards is evident in such a simple example. But the structured statements of the sections of the Internal Revenue Code are more complex than implications that have just one-element antecedents. How shall edge-punched cards work with these more complex statements as premises? Observe, first, that although the antecedent of a given structured statement may have many conditions that must be fulfilled in various combinations as specified by a complex network of logical relationships between the conditions in order for some given consequence to follow, there are only three logical relationships involved: conjunction, disjunction, and negation. If each of these three can be dealt with in some mechanical fashion by means of the edge-punched cards, then the structured statements of the Internal Revenue Code can be handled mechanically whatever their complexity.

Consider next a premise that is an implication with a conjunction as its antecedent. Does P4 follow from the set of premises P1,
P2, and \( \rightarrow P2-P3 \rightarrow P4 \)? In order to answer this question by means of the set of edge-punched cards, our strategy is to determine and inspect the subset of the total set of cards that (1) is compatible with each of the given premises and (2) contains the most members.

Since the first premise is P1, we needle set 1 (the total set that contains all sixteen of the cards) in hole-P1. The result is that cards 0–7 hang on \( H(0–7) \) and cards 8–15 drop out \( D(8–15) \). This operation and its results will be described in tabular form as follows:

<table>
<thead>
<tr>
<th>STEP</th>
<th>OPERATION</th>
<th>RESULTS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Needle set 1 through hole-P1</td>
<td>set 2: ( H(0–7) ) set 3: ( D(8–15) )</td>
</tr>
</tbody>
</table>

The eight cards in set 3 comprise the set that is compatible with the first premise. Next we determine those contained in set 3 that are compatible with the second premise. Since the second premise is P2, we needle set 3 through hole-P2.

| 2.   | Needle set 3 through hole-P2 | set 4: \( H(8–11) \) set 5: \( D(12–15) \) |

The four cards in set 5 comprise the set that is compatible with both the first and second premises. Next we determine those contained in set 5 that are compatible with the third premise. Since the third premise is \( \rightarrow P2-P3 \rightarrow P4 \), the combination of P2, P3, and NP4 is not permitted and must be eliminated. The next three operations accomplish this.

| 3.   | Needle set 5 through hole-P2 | set 6: \( H(\_\_) \) set 7: \( D(12–15) \) |
| 4.   | Needle set 7 through hole-P3 | set 8: \( H(12, 13) \) set 9: \( D(14, 15) \) |
| 5.   | Needle set 9 through hole-P4 | set 10: \( H(14) \) set 11: \( D(15) \) |

The subset of set 5 that is incompatible with the third premise is set 10, which is comprised of card 14 alone. The subset of set 5 that is compatible with the third premise, and therefore compatible with all three premises, is the subset comprised of set 5 with card 14 eliminated (the set comprised of cards 12, 13, and 15). If the
set of cards compatible with all of the premises are stacked to form a deck, the deck will look like this:

```
P1  P2  P3  P4
```

This set of cards indicates that the answer to the question of whether P4 follows from the set of premises P1, P2, and \( \rightarrow P2-P3 \rightarrow P4 \), is negative. This is so because it is not the case that there is a 'U' in hole-P4 of every card in the set that is compatible with all three premises. Card 12 has an 'O' in hole-P4.

The next example will illustrate the operations on the edge-punched cards with a premise that contains an antecedent involving conjunction and negation. Does P4 follow from the set of premises P1, P2, NP3, and \( \rightarrow P1-NP3 \rightarrow P4 \)? The operations and results would be as follows:

<table>
<thead>
<tr>
<th>STEP OPERATION</th>
<th>RESULTS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Needle set 1 through hole-P1</td>
<td>set 2: H(0–7) set 3: D(8–15)</td>
</tr>
<tr>
<td>3. Needle set 5 through hole-P3</td>
<td>set 6: H(12, 13) set 7: D(14, 15)</td>
</tr>
<tr>
<td>4. Needle set 6 through hole-P1</td>
<td>set 8: H( ) set 9: D(12, 13)</td>
</tr>
<tr>
<td>5. Needle set 9 through hole-P3</td>
<td>set 10: H(12, 13) set 11: D( )</td>
</tr>
</tbody>
</table>

The subset of set 1 that is compatible with all four premises is the set comprised of card 13. Every card in the subset compatible with the premises has a 'U' in hole-P4. Therefore, P4 does follow from the four premises P1, P2, NP3, and \( \rightarrow P1-NP3 \rightarrow P4 \).

The next example will illustrate the operations on the edge-punched cards with a premise that contains an antecedent involving conjunction, negation, and disjunction. Does P4 follow from
the set of premises $P_1$, $NP_3$, and $\neg P_1$ and $NP_3 \Rightarrow P_4$.

The operations and results would be as follows:

<table>
<thead>
<tr>
<th>STEP</th>
<th>OPERATION</th>
<th>RESULTS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Needle set 1 through hole-$P_1$</td>
<td>set 2: $H(0-7)$ set 3: $D(8-15)$</td>
</tr>
<tr>
<td>2.</td>
<td>Needle set 3 through hole-$P_3$</td>
<td>set 4: $H(8, 9, 12, 13)$ set 5: $D(10, 11, 14, 15)$</td>
</tr>
<tr>
<td>3.</td>
<td>Needle set 4 through hole-$P_1$</td>
<td>set 6: $H(\ )$ set 7: $D(8, 9, 12, 13)$</td>
</tr>
<tr>
<td>4.</td>
<td>Needle set 7 through hole-$P_2$</td>
<td>set 8: $H(8, 9)$ set 9: $D(12, 13)$</td>
</tr>
<tr>
<td>5.</td>
<td>Needle set 8 through hole-$P_3$</td>
<td>set 10: $H(8, 9)$ set 11: $D(\ )$</td>
</tr>
<tr>
<td>6.</td>
<td>Needle set 12 through hole-$P_4$</td>
<td>set 13: $H(8, 12)$ set 14: $D(9, 13)$</td>
</tr>
</tbody>
</table>

Set 4 is the subset of set 1 that is compatible with the premises $P_1$ and $NP_3$. The next set of operations will eliminate those cards that are incompatible with the third premise (those cards on which the combination of $P_1$, $(P_2 \lor NP_3)$, and $NP_4$ occurs).

Set 12 is comprised of all the cards in set 9 plus all the cards in set 10 (cards 8, 9, 12, and 13).

The set to be eliminated from set 4 is set 13, leaving cards 9 and 13 to comprise the set that is compatible with all three of the premises. When stacked in a deck this compatible set looks like this:

```
[0 0 0 U]
P1 P2 P3 P4
```

Set of Cards

9, 13

Since hole-$P_4$ does have a 'U' on all of the cards in the compatible deck, $P_4$ does follow from the three premises $P_1$, $NP_3$, and $\neg P_1$ and $NP_3 \Rightarrow P_4$. 

$\neg P_1 \Rightarrow P_2$
Since the structured statement versions of the sections of the Internal Revenue Code are merely implications that have antecedents with more complex combinations involving these three logical relationships of conjunction, negation, and disjunction, the deduction involved in concluding what consequences follow from (1) a set of specified fulfilled conditions and (2) a set of norms can all be done mechanically in the manner illustrated by means of these simple examples. To the extent that the lawyer is freed from this routine task, he can then spend a greater amount of time on the much more important and difficult task of judging just what events and states of affairs will be held by a community decision maker to fulfill what conditions.

To indicate that even a simple mechanical device can handle problems that require some effort on the part of a human thinker, the reader is invited to test his intuitions as to whether it can be validly concluded from the following three premises that mathematical logic will probably be taught in law schools during the next decade.

S 1 If

1. it is not so that lawyers can easily learn mathematical logic,
then
2. there is a group of professional communicators that will be badly deficient in a required skill in the near future.

S 2 If

1. there is a group of professional communicators that will be badly deficient in a required skill in the near future,
then
2. mathematical logic will probably be taught in law schools during the next decade, and
3. lawyers can easily learn mathematical logic.

S 3 It is not so that both
1. lawyers can easily learn mathematical logic, and
2. it is not so that mathematical logic will probably be taught in law schools during the next decade.

Let us use the following representations:

P1 Lawyers can easily learn mathematical logic.
P2 There is a group of professional communicators that will be badly deficient in a required skill in the near future.

P4 Mathematical logic will probably be taught in law schools during the next decade.

The three premises can then be represented by ‘>—NP1——→P2’, ‘>—P2——→P4-P1’, and 'N(P1-NP4)'. To find the subset compatible with all three premises, we proceed as follows:

<table>
<thead>
<tr>
<th>STEP OPERATION</th>
<th>RESULTS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Needle set 1 through hole-P1</td>
<td>set 2: H(0-7)</td>
</tr>
<tr>
<td>2. Needle set 2 through hole-P2</td>
<td>set 3: D(8-15)</td>
</tr>
<tr>
<td>3. Needle set 6 through hole-P2</td>
<td>set 4: H(0-3)</td>
</tr>
<tr>
<td>4. Needle set 8 through hole-P4</td>
<td>set 5: D(4-7)</td>
</tr>
<tr>
<td>5. Needle set 11 through hole-P2</td>
<td>set 7: H(8-11)</td>
</tr>
<tr>
<td>6. Needle set 13 through hole-P1</td>
<td>set 8: D(4-7, 12-15)</td>
</tr>
<tr>
<td>7. Needle set 16 through hole-P1</td>
<td>set 9: H(4, 6, 12, 14)</td>
</tr>
<tr>
<td>8. Needle set 18 through hole-P4</td>
<td>set 10: D(5, 7, 13, 15)</td>
</tr>
</tbody>
</table>

Set 6, which is the set of cards compatible with the first premise, is comprised of all the cards in set 1 that are not in set 4 (cards 4-15).

Set 11, which is the set of cards compatible with the first premise and the >—P2——→P4 half of the second premise, is comprised of all the cards in set 6 that are not in set 9 (cards 5, 7-11, 13, and 15).

Set 16, which is the set of cards compatible with the first and second premises, is comprised of all the cards in set 11 that are not in set 14 (cards 8-11, 13, and 15).

Set 21, which is the set of cards compatible with all three premises, is comprised of all the cards in set 16 that are not in set 19 (cards
When this compatible set is stacked in a deck, it looks like this:

```
U 0 0 U
P1 P2 P3 P4
```

Set of Cards
9, 11, 13, 15

Because the hole-P4 has a 'U' in every card in the compatible set, we know that the consequence represented by 'P4' follows from the three premises represented by ‘>-----NP1-----P2’, ‘-----P2------P4-P1’, and ‘N(P1-NP4)’.

In his paper on A Preliminary Design for Coding Statutes to Punched Cards, which describes a different way of using edge-punched cards for doing the kind of deduction illustrated above, Helgeson has catalogued some of the kinds of questions to which answers may be found mechanically when the statute is systematically organized; he has also shown how to find the answers to such questions. Briefly these are:

Q1 Given that a specified set of conditions has been fulfilled, what consequences (if any) follow?

Q2 Given that a certain consequence does not follow from a specified set of fulfilled conditions, what additional conditions must be fulfilled in order for that consequence to follow?

Q3 Given a certain fulfilled condition, what are all the consequences that have that condition included in their antecedents?

Q4 Given a certain fulfilled condition and all the consequences that have that condition included in their antecedents, what are the other conditions that must be fulfilled for each of those consequences to follow?

Q5 Given that a certain condition is not fulfilled, what are all the consequences that are prevented from following?

Q6 Given a certain consequence as a desirable goal, what alternative sets of conditions must be fulfilled for that consequence to follow?

Q7 Given that it is desirable as a goal that a certain conse-
sequence be avoided, the fulfillment of what set of conditions will assure this?

Q8 Given a specified set of fulfilled conditions, does a certain consequence follow?

Q9 Given a specified set of fulfilled and unfulfilled conditions and a certain consequence that does not follow from that set of conditions, which conditions (if any) are preventing that consequence from following?

Q10 Given that a specified set of conditions are fulfilled and that another condition is also fulfilled, what additional consequences will follow if that other condition is not fulfilled?

Q11 Given that from a specified set of fulfilled conditions a certain consequence follows, which of those conditions can be unfulfilled (if any) and still have that consequence follow?

Lawyers can also be provided with additional manual methods for finding information that they need by having a list prepared of all the conditions mentioned in a given statute that is so organized and a list of all the consequences mentioned, along with indices to such lists.

If lawyers would welcome such assistance as can be provided by automatic devices and the simplification of their tasks that can be achieved by organizing the tax materials in a more systematic form, then perhaps it is not inappropriate to suggest that some commercial or governmental organization ought to begin thinking about providing the service of organizing the materials in a suitable manner. If such help is not welcome, perhaps all of us ought to wonder why.