FINDING ORDER IN THE MORASS: THE THREE REAL JUSTIFICATIONS FOR PIERCING THE CORPORATE VEIL

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Few doctrines are more shrouded in mystery or litigated more often than piercing the corporate veil. We develop a new theoretical framework that posits that veil piercing is done to achieve three discrete public policy goals, each of which is consistent with economic efficiency: (1) achieving the purpose of an existing statute or regulation; (2) preventing shareholders from obtaining credit by misrepresentation; and (3) promoting the bankruptcy values of achieving the orderly, efficient resolution of a bankrupt’s estate. We analyze the facts of veil-piercing cases to show how the outcomes are explained by our taxonomy. We demonstrate that a supposed justification for veil piercing—undercapitalization—in fact rarely, if ever, provides an independent basis for piercing the corporate veil. Finally, we employ modern quantitative machine learning methods never before utilized in legal scholarship to analyze the full text of 9,380 judicial opinions. We demonstrate that our theories systematically predict veil-piercing outcomes, that the widely invoked rationale of “undercapitalization” of the business poorly explains these cases, and that our theories most closely reflect the actual textual structure of the opinions.

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INTRODUCTION

The doctrine of piercing the corporate veil is shrouded in misperception and confusion. On the one hand, courts understand the fact that the corporate form is supposed to be a juridical entity with the characteristic of legal "personhood." As such, courts acknowledge that their equitable authority to pierce the corporate veil is to be exercised "reluctantly" and "cautiously." Similarly, courts also recognize that it is perfectly legitimate to create a corporation or other form of limited liability business organization such as an LLC "for the very purpose of escaping personal liability" for debts incurred by the enterprise.2

Apparently inconsistent with the "limited liability" nature of the corporate enterprise, the list of justifications for piercing the corporate veil is long, imprecise to the point of vagueness, and less than reassuring to investors and other participants in the corporate enterprise interested in knowing with certainty what the limitations are on the scope of shareholders' personal liability for corporate acts. For example, veil piercing may be done where the corporation is the mere "alter ego" of its shareholders; where the corporation is undercapitalized; where there is a failure to observe corporate formalities; or where the corporate form is used to promote fraud, injustice, or illegalities.3

In this Article we argue that there is a rational structure to the doctrine of corporate veil piercing not only in theory but in practice as well. Our idea is that, despite the fact that courts are inarticulate to the point of incoherence in their reasoning in particular "piercing" cases, a rational taxonomy can be derived from this morass.

Our thesis is simple. We begin with the observation that the so-called "doctrine" of piercing the corporate is not a doctrine at all. It is a remedy. Like any good remedy, of course, the corporate veil is

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3 See Baatz v. Arrow Bar, 452 N.W.2d 158, 141 (S.D. 1990).
pierced in order to achieve discrete, specific policy objectives. As such, our task is to discover and articulate what those policy objectives are. Our taxonomy consists of the list of the policy objectives that justify ignoring the corporate form. In the case of piercing, one reason that confusion and incoherence reigns is that the corporate veil is pierced in order to accomplish three separate and largely unrelated, albeit legitimate, policy objectives.

The entire universe of piercing cases can be explained as judicial efforts to remedy one of the following three problems. While some of these problems previously have been identified, this is the first Article to identify all of the economic and policy problems that piercing attempts to ameliorate. It is the first to present a taxonomy that can explain all of the decisions in this area and that can be used methodologically to evaluate the quality of piercing decisions.

First, courts pierce the corporate veil as a tool of statutory application, in the sense that piercing the corporate veil is done in order to bring corporate actors’ behavior into conformity with a particular statutory scheme, such as social security or state unemployment compensation schemes. For example, as explained in detail below, sometimes the court will ignore the corporate form in order to accomplish the specific legislative goal of a government benefit program that distinguishes between owners and employees. And of course, sometimes the court will respect the corporate form where doing so is necessary to reach a result that is consistent with a particular state or federal statutory scheme.

Second, courts also pierce in order to remedy what appears to be fraudulent conduct that does not satisfy the strict elements of common law fraud. Specifically, courts pierce as a remedy for “constructive fraud” in the contractual context. Simply put, if a court becomes convinced that a shareholder or other equity investor has, by words or actions, led a counterparty to a contract to believe that an obligation is a personal liability rather than (or in addition to) a corporate debt, then courts sometimes will use a piercing theory to impose liability on the individual shareholder rather than a fraud theory. As one court famously has observed, “[F]raud or something like it is required” to pierce the corporate veil whether under federal, Delaware, or Oklahoma common law.\(^4\)

The third ground on which courts pierce the corporate veil that we identify is the promotion of what we term accepted “bankruptcy values.” In particular, courts will disregard the corporate form in order to prevent fraudulent conveyances and preferential transfers. The

goal of corporate bankruptcy law is to maximize the value of an insolvent company for the benefit of all of the creditors. An important element in accomplishing this goal is to resolve the collective action problem facing a corporation’s creditors, who in the absence of such rules as the automatic stay, which prevents creditors from grabbing the assets of a company after it has filed for bankruptcy, have incentives to race for relief in order to get a jump on other creditors.

Similarly, of course, bankruptcy law strives to achieve an orderly disposition of the debtors’ assets, either through corporate reorganization or liquidation. One way that bankruptcy law achieves these goals is by preventing shareholders from transferring corporate assets to themselves or to particular favored creditors ahead of other creditors in times of acute economic stress. This result is accomplished in the context of a formal bankruptcy proceeding by invoking the doctrine of equitable subordination as well as by the bankruptcy trustee’s power to avoid and set aside preferential transfers and fraudulent conveyances. Outside of bankruptcy (and sometimes in the context of bankruptcy proceedings as well), the goal of eliminating opportunism by companies in financial distress is accomplished by disregarding the corporate form.

All of the piercing cases can be explained as an effort to accomplish one of these three goals: (1) achieving the goals of a particular regulatory or statutory scheme; (2) avoiding fraud or misrepresentation by shareholders trying to obtain credit; and (3) promoting the bankruptcy value of eliminating favoritism among claimants to the cash flows of a firm.

Thus it is our view that all of the standard litany for justifications for disregarding the corporate form, which include failure to observe corporate formalities, undercapitalization, alter ego, mere instrumentality, ownership of all or most of the stock in the company, payment of dividends, failure to pay dividends, etc., are mere proxies for one of the three core reasons for piercing described above.

Our thesis is descriptive and empirically verifiable. It is not directly normative, though we do make some normative observations, and we believe that the taxonomy we develop here will facilitate the formulation of normative judgments about the desirability of piercing in the ways that we describe below. We argue that veil-piercing cases, if they are correctly decided, will only reach investors’ assets if doing so accomplishes one of the three goals for piercing that we identify.

6 Former dean of Harvard Law School Robert Clark was the first to recognize the connection between piercing the corporate veil and the doctrine of equitable subordination and the law of fraudulent conveyance. See Robert Clark, Corporate Law 84–85 (1986).
We demonstrate that our theory consistently explains the results in the leading cases on piercing the veil. Significantly, we find no piercing cases in which a court pierces the corporate veil solely because a corporation is undercapitalized. This finding is consistent with the fact that legislatures permit thinly capitalized firms to engage in business and generally do not require that companies be well-capitalized in order to be formed.

Moreover, we find that, although courts do invoke the mantra of undercapitalization to justify a determination to pierce the corporate veil, we find that, in each case, there are other justifications for veil piercing that are consistent with our taxonomy.

We test our theory systematically by applying machine learning and automated text analysis methods to classify 9,380 federal and state cases mentioning veil piercing or disregarding the corporate form. We show that the three goals we have identified are a superior predictor of actual veil-piercing decisions than the largely incoherent doctrines espoused by the courts. We also show that undercapitalization is actually a particularly poor predictor of veil-piercing outcomes. Most significantly, in our view, we find that the application of topic modeling demonstrates that the distribution of ideas in the text of these opinions tracks our theories more or less precisely.

The confusion and incoherence of the law of piercing the corporate veil is compounded by the fact that because judges do not explicitly recognize these policy objectives, their justifications for piercing the corporate veil are incoherent. In our view, judges generally reach the correct results in the cases they decide. Because the judges (and clerks) writing these decisions do not understand the theory (policy rationale) on which their decisions are based, these results most likely are based on simple intuition. As a consequence of this intellectually vapid reasoning by intuition, legal opinions are full of vague assertions about nonsensical justifications such as "observing corporate formalities" and broad generalizations such as "achieving justice."

As one of us has observed, "in no other area are courts more prone to decide real life disputes by characterization, epithet, and metaphor: 'alter ego,' 'instrumentality,' 'sham,' 'subterfuge,' or 'tool,' to select a few," rather than on carefully articulated reasons.

As Phillip Blumberg, the most prolific scholar in the piercing space, accurately observed, the law of piercing the corporate veil is:

jurisprudence by metaphor or epithet. It does not contribute to legal understanding because it is an intellectual construct, divorced from business realities . . . . [C]ourts state that the corporate entity

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is to be disregarded because the corporation is, for example, a mere "alter ego." But they do not inform us why this is so, except in very broad terms that provide little general guidance. As a result, we are faced with hundreds of decisions that are irreconcilable and not entirely comprehensible. Few areas of the law have been so sharply criticized by commentators.  

Courts, on the other hand, often appear unconcerned by the vagueness of the doctrines they are formulating. One court observed that a "guiding concept behind . . . veil-piercing cases is the need for the court to 'avoid an over-rigid preoccupation with questions of structure . . . and apply the preexisting and overarching principle that liability is imposed to reach an equitable result.'"  

In Part I of our Article we explain the current state of the law and show the analytical shortcomings of current jurisprudence. In Part II we present and defend our own taxonomy for explaining the cases in light of a qualitative discussion of leading veil-piercing cases. In Part III we present the results of our quantitative study employing automated content analysis. A conclusion follows.

I
THE CURRENT MORASS

A. Limited Liability and Veil-Piercing Doctrine

Analytically, the concept of limited liability is a simple, albeit profound, implication of the basic concept in corporate law that corporations (and other entities such as limited liability companies and limited partnerships) are distinct juridical entities separate and apart from their creditors, shareholders, directors, and other constituencies. As a consequence of this legal separateness, corporations and similar business entities can enter into contracts; sue and be sued; be responsible for paying taxes and complying with laws and regulations; and internalize both the benefits and the burdens of these attributes separate and distinct from other constituencies, including shareholders. In other words, limited liability is an implication of the notion that a complete separation of identities exists between investors and the corporations in which they have invested.

To the extent that the relationship between a corporation and its investors is purely contractual, that contract need not, and generally does not, require that investors face personal liability to the corporation's creditors. Similarly, of course, the existence of this corporate separateness implies that investors in the corporation—whether they

are equity investors or debt investors—have only an attenuated relationship with the people and companies with whom the corporation interacts. And that relationship does not include personal responsibility or liability for the debts of the corporation on the part of investors.

Thus, limited liability derives from the concept of corporate separateness, and corporate separateness is now firmly ingrained in our legal culture and in the sensibilities of lawyers and investors. This has not always been the case. Early corporations in the United States often did not provide for limited liability for shareholders. As Morton Horwitz has observed, "truly limited shareholder liability was far from the norm in America even as late as 1900." As a theoretical matter, the concept of complete corporate separateness was required in order to make the idea of limited liability defensible, or at least coherent.

The concept of corporate separateness is now firmly ensconced in the legal culture. The corollary of this concept is limited liability for shareholders. Because the corporation is solely liable for the debts of the corporation, it follows inexorably that others, including claimants on the cash flows of the business such as shareholders, are not personally liable for the debts that the corporation has incurred. Limited liability is considered "the primary benefit of the corporate form," and the protections for shareholders have been described, somewhat inaccurately, as "both unqualified and universal" for corporations (and other entities such as limited liability companies and limited liability partnerships) of every size, shape, and description.

Whenever a corporation is unable fully to meet its creditors' demands for payment, such creditors will find it in their interest to try to obtain payment of the corporation's obligations to them from other parties, and the company's shareholders generally are the most attractive target. Creditors in this situation often will try to persuade a court to pierce the corporate veil or to disregard the corporate form.

The lack of a coherent theory or workable algorithm either for determining when the corporate veil should be pierced or for predicting when it will be pierced results in a situation in which creditors lack the disincentive to sue—and defendants lack the incentive to settle—

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12 See id. at 185–86 (discussing early theories of the corporation and the possibility of limited liability under them).
14 Id. at 340.
that clear legal rules provide. The ostensible standards articulated by courts to justify piercing vary from jurisdiction to jurisdiction, and the articulated doctrinal standards are "often characterized by ambiguity, unpredictability, and even a seeming degree of randomness."  

Large swaths of veil-piercing doctrine make no sense and do not promote any sensible policy goals such as limiting opportunistic risk-taking. Piercing occurs sporadically, and it is difficult to predict when piercing will occur.  

The often-articulated maxim that courts pierce the corporate veil in order to "do justice" creates the erroneous impression that this is an area of the law in which judges are unconstrained by rules and are more or less free to impose remedies that fit their own internal views of fairness. As Robert Clark observed in his treatise, "the courts usually forgo any sustained attempt at a remedial theory or even a coherent exposition of the basis of liability, although descriptive summaries are occasionally attempted." A major point of this Article is that judges have in fact decided veil-piercing cases in a highly disciplined and structured way when one analyzes the actual outcomes of the cases in isolation from the reasoning displayed in the decisions themselves.

A sure sign of the state of confusion in this area of law is the existence of incoherent and inconsistent multifactor tests. For example, occasionally, a failure to pay dividends is identified as a justification for piercing the corporate veil, while in other contexts the payment of dividends is itself identified as a justification for piercing the corporate veil. Obviously, both of these justifications cannot si-

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16 See id. at 506–07; see also Frank H. Easterbrook & Daniel R. Fischel, Limited Liability and the Corporation, 52 U. Chi. L. Rev. 89, 89 (1985) (calling veil piercing "rare, severe, and unprincipled").
18 Clark, supra note 6, at 72.
19 See Riggins v. Dixie Shoring Co., 592 So. 2d 1282, 1283 (La. 1992) (Dennis, J. concurring in the denial of rehearing) ("Some of the many factors which may properly be considered [in veil-piercing cases] include: . . . non-payment of dividends.").
multaneously be valid. Among the other factors that courts claim to consider when they pierce the corporate veil are:

- significant undercapitalization of the business entity (capitalization requirements vary based on industry, location, and specific company circumstances);\(^\text{21}\)
- failure to observe corporate formalities in terms of behavior and documentation;\(^\text{22}\)
- intermingling of activities or assets of the corporation and of the shareholder;\(^\text{23}\)
- treatment by an individual of the assets of the corporation as his/her own;\(^\text{24}\)
- failure to pay dividends;\(^\text{25}\)
- siphoning of corporate funds by the dominant shareholder(s), through, inter alia, the payment of dividends;\(^\text{26}\)
- nonfunctioning corporate officers and/or directors;\(^\text{27}\)
- absence or inaccuracy of corporate records;\(^\text{28}\)
- overlap of corporate records, functions, or personnel;\(^\text{29}\)
- use of the corporation as a “facade” for personal dealings of the dominant shareholder(s), the “alter ego theory”;\(^\text{30}\)


\(^{22}\) See id.

\(^{23}\) See, e.g., *Cancun Adventure Tours, Inc. v. Underwater Designer Co.*, 862 F.2d 1044, 1047-48 (4th Cir. 1988) (“When substantial ownership is combined with other factors, such as commingling of corporate and personal assets and diversion of corporate funds to the dominant shareholder, a court may peer behind the corporate veil.”); *My Bread Baking Co. v. Cumberland Farms, Inc.*, 233 N.E.2d 748, 751-52 (Mass. 1968) (discussing how the separate identities of affiliated corporations may be disregarded when “there is a confused intermingling of activity of two or more corporations engaged in a common enterprise with substantial disregard of the separate nature of the corporate entities, or serious ambiguity about the manner and capacity in which the various corporations and their respective representatives are acting”); see also Peter B. Oh, *Veil-Piercing*, 89 TEX. L. REV. 81, 133 tbl.12 (2010) (reporting that fraud/misrepresentation, injustice/unfairness, domination, commingling of assets, and undercapitalization, are the most popular rationales for veil piercing).

\(^{24}\) See, e.g., *Cahaly v. Benistar Prop. Exch. Trust Co.*, 864 N.E.2d 548, 557 n.15 (Mass. App. Ct. 2007) (“[Company] funds were used by [one of the individual defendants] to carry out ‘his personal penchant for risky option trading on the stock market.’”).


\(^{26}\) See, e.g., *Keffer v. H.K. Porter Co.*, 872 F.2d 60, 65 (4th Cir. 1989) (listing factors relevant to the decision to pierce the corporate veil as including, inter alia, “siphoning of the corporation’s funds”).

\(^{27}\) See, e.g., id. (listing factors relevant to the decision to pierce the corporate veil as including, inter alia, “non-functioning of officers and directors”).

\(^{28}\) See, e.g., *Cahaly*, 864 N.E.2d at 557.


\(^{30}\) See, e.g., *H.K. Porter Co.*, 872 F.2d at 65 (holding that “the fact that the corporation is merely a facade for the operation of the dominant stockholder or stockholders” is relevant to the veil-piercing decision); *Galgay v. Gangloff*, 677 F. Supp. 295, 298 (M.D. Pa. 1987) (listing the alter ego doctrine as an exception to the general rule against shareholder
• failure to maintain arm's length relationships with related entities;\textsuperscript{31}
• use of the corporation to promote fraud, injustice, or illegalities;\textsuperscript{32} and
• payment by the corporation of individual obligations.\textsuperscript{33}

Among the many problems with the current approach is its failure to draw any sort of logical link between the articulated test or factor and any discernible public policy objective. For example, while it often is claimed that piercing the corporate veil is required to prevent fraud,\textsuperscript{34} this is not the case. Where there is fraud, a cause of action for fraud can be brought directly, and there is no reason to pierce the corporate veil in order to prevent injustice. Similarly, where there is no fraud, it hardly makes sense to impose liability as a means of preventing some other, unrelated fraud in the future.

Likewise, it seems nothing short of bizarre to impose liability on a shareholder on the grounds that the corporation has not been scrupulous about keeping minutes or other records unless there is some connection between the sloppy or nonexistent record-keeping and the harm to the plaintiff, which generally there is not.

Indeed, the rather popular idea of disregarding the corporate form on the grounds that the corporation is undercapitalized is similarly problematic.\textsuperscript{35} It is inconsistent with the basic concept that a corporation can be formed for the express purpose of avoiding personal liability to impose an ex post condition on personal liability that has nothing to do with the shareholders' own behavior. As a practical matter, the lack of minimum capital requirements, and the existence of thousands of corporations that have been formed and are being operated for long periods of time without any capital at all, is only consistent with the theory that capital is not required.\textsuperscript{36}

\textsuperscript{32} See, e.g., Baatz v. Arrow Bar, 452 N.W.2d 138, 141 (S.D. 1990) (listing factors relevant to the decision to pierce the corporate veil as including, inter alia, "use of the corporation to promote fraud, injustice, or illegalities").
\textsuperscript{33} See, e.g., id. (listing factors relevant to the decision to pierce the corporate veil as including, inter alia, "payment by the corporation of individual obligations").
\textsuperscript{34} See, e.g., DeWitt Truck Brokers, Inc. v. W. Ray Flemming Fruit Co., 540 F.2d 681, 684 (4th Cir. 1976) (discussing cases where courts have cited fraud as justification for veil piercing).
\textsuperscript{35} See Oh, supra note 23, at 133 tbl.12 (reporting empirical findings that undercapitalization is among the most popular rationales for veil piercing); Thompson, supra note 29, at 1063 tbl.11 (same).
We have found that courts do not pierce the corporate veil unless there is a cogent policy justification for doing so. We have identified three such cogent policy reasons for piercing. We recognize that sometimes courts articulate rationales for their decisions in piercing cases that ostensibly vary from ours. For example, courts sometimes defend piercing the corporate veil because a company: (a) has failed to follow corporate formalities, (b) is the "alter ego" of its shareholders, or (c) is undercapitalized. We find however that when one looks beyond the ritualistic incantation of these vague talismanic mantras, when the veil is actually pierced, invariably the real reason for doing so is that piercing achieves one of the three policy justifications identified in this Article.

We refer to certain ostensible justifications for piercing the corporate veil as "talismanic mantras" where there is no logical link or nexus between the justification and a concrete economic rationale or public policy justification for veil piercing. For example, piercing the corporate veil for failing to observe corporate formalities such as holding directors' meetings or keeping minutes makes no sense. It is like imposing liability on a person because he did not wear a tie or keep a napkin in his lap while eating. On the other hand, where the failure to keep records is so profound that one cannot utilize such records to determine which assets legitimately belong to the corporation and which legitimately belong to its shareholders, then piercing is appropriate to prevent the unfair and strategic abuse of creditors by unilaterally categorizing assets as belonging to the shareholder (and thus unavailable to creditors) when there is no legitimate basis for doing so. Thus, when "failure to observe corporate formalities" is cited as a justification for piercing, the piercing generally is done to achieve one of the concrete policy goals articulated here, such as promoting bankruptcy values or preventing "[f]raud or something like it."

Indeed, as Franklin Gevurtz has noted, opportunistically establishing a corporation to exploit the benefits of a statutory scheme, i.e., to engage in a type of statutory arbitrage, is another type of conduct that is "like" fraud, and therefore justifies piercing even though it does not rise to the level of classic common law fraud.

A similar form of analysis applies for the invocation of the alter ego doctrine. Clearly when the activities of a corporation and its shareholders are so intertwined that it is impossible to distinguish one from the other, i.e., to determine where the corporation's assets and

activities stop and where the shareholders' assets and activities begin, then piercing is required to achieve the central policy objectives of bankruptcy law: facilitating the fair and orderly resolution of a bankrupt's estate and avoiding transfers to shareholders that undermine the legitimate, contractually determined expectations of the creditors.

Turning to the issue of undercapitalization, we first draw attention to the often-overlooked fact that piercing a corporation's veil on the sole grounds that the corporation is (or was) undercapitalized is fundamentally inconsistent with two bedrock tenets of corporate law. First, piercing the corporate veil on the grounds of undercapitalization is inconsistent with the legislative determination that corporations can be properly formed without any capital whatsoever. This legislative principle is reflected in state statutes in Delaware and other states permitting corporations to be formed without equity contribution from their founders.39

Second, piercing on the grounds that a corporation is undercapitalized is inconsistent with the well-known policy determination that shareholders can form corporations for the specific purpose of avoiding personal liability.40 The assertion that a corporation must maintain enough stockholders' equity to satisfy the claims of creditors is inconsistent with the notions of asset partitioning and of limiting shareholders' liability to the amount of their original capital contributions (if any).

B. Scholarly Attempts to Chart a Path Forward

The academic literature on piercing the corporate veil largely suffers from the same analytical incoherence as the doctrine itself. Numerous scholars have attempted to make sense of the confusing and contradictory applications found in the case law by offering various normative and descriptive theories as well as empirical studies. Unfortunately, these important works have yet to provide a comprehensive taxonomy that systematically explains existing case law.

In a classic treatment of the topic, Frank Easterbrook and Daniel Fischel build on Henry Manne's argument that limited liability facili-


40 In fact, the "primary reason" for using the corporate (or LLC) form of business organization "is to protect the owners and management from personal liability for claims against the business." Valerio Giannini, Governance and Records in Closely-Held Corporations, available at http://riskarticles.com/wp-content/uploads/2013/02/Governance-and-Records-in-Closely-Held-Companies.pdf. See generally Easterbrook & Fischel, supra note 16.
tates portfolio diversification by emphasizing that limited liability serves to more efficiently allocate risk between creditors and shareholders: rather than imposing the entirety of the monitoring, information, and coordination costs on shareholders, limited liability places those burdens on creditors, who are often better equipped to bear them. Veil-piercing cases can be understood as judicial recognitions of the inapplicability of this efficiency rationale in settings such as close corporations (where the benefits of portfolio diversification are few), parent-subsidiary corporations (no unlimited liability for shareholders), tort creditors, and undercapitalization (to reduce socially excessive risk-taking).

The problem with Easterbrook and Fischel's approach is that it provides no clear decision rule for courts. What is the line between a corporation that is sufficiently closely held to warrant veil piercing and one that is not? Should subsidiary corporations always lack limited liability? Courts have refused to go down those paths, opting instead for multifactor tests that preserve judicial discretion. Indeed, it is difficult to see how any of these settings inherently tips the balance toward excessive risk externalization toward creditors. This suggests that Easterbrook and Fischel's theory provides only an ex post justification for veil-piercing cases rather than an ex ante rule that courts can reliably implement. A more comprehensive account of veil-piercing cases is necessary to understand the actual scope of the doctrine.

In perhaps the most well-known study on piercing the corporate veil, Robert Thompson attempted to shed light on the true scope of the doctrine by empirically examining approximately 1,600 cases from 1930 to 1985. Thompson found that the empirical data did not comport fully with theoretical intuitions. In particular, courts were more likely to pierce the veil when individual shareholders were involved than for corporate shareholders, and courts pierced the veil more often in contract than in tort cases. These findings stand in contradiction to Easterbrook and Fischel's theory.

Thompson's study provides valuable insights by summarizing and classifying the 1,600 cases in his dataset. But there are several methodological shortcomings to his approach. As others have noted, case counting alone does not permit distinguishing which factors are more strongly associated with an outcome than others. Thompson's effort...

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42 Id. at 109-12.
43 Thompson, supra note 29, at 1036.
44 Id. at 1068.
to identify which veil-piercing rationales are mentioned by courts sheds important light on these cases, but mentioning a rationale is not the same as relying on it. To merely say that something is mentioned in a "laundry list" of factors does not provide much insight. We build on Thompson’s work by looking at how stated veil-piercing rationales actually influence courts, i.e., by evaluating which are systematically associated with veil-piercing outcomes. Thus, our study permits determining which factors are actually driving veil-piercing decisions.

Indeed, we are not the first to attempt to determine which factors are systematically associated with veil-piercing outcomes. John Matheson conducted an extensive empirical study using multiple logistic regressions to identify which case characteristics predict a veil-piercing outcome. More recently, Peter Oh examined nearly 3,000 veil-piercing cases and found that fraud claims are the best predictor of a piercing decision and, contrary to Thompson, concluded that tort claims were more successful than contract claims. Interestingly, Oh also found that undercapitalization rationales were mentioned at similar proportions in contract and tort cases, which makes less sense with respect to the latter as contract creditors had the opportunity obtain an express guarantee from the principals of an undercapitalized firm.

A shortcoming of all of these studies is the use of mechanical coding to identify veil-piercing factors, which is subject to substantial subjectivity and arbitrariness. Coding schemes—indeed, quantitative analysis more generally—necessarily reflect an imperfect approximation of the qualitative complexity of each case. But using mechanical coding to identify determinants of veil piercing is particularly imprecise because it places substantial discretion in the hands of human coders, whose application of judgment can vary between individuals and even from case to case by the same individual.

Indeed, the contradictory findings of Peter Oh and Robert Thompson with respect to tort versus contract cases serve as a useful case in point. It is likely that Oh’s reclassification of many contract cases under a new category of “fraud” is responsible for explaining this difference, which Oh tacitly acknowledges. Regardless of whether fraud falls into contract or tort as a doctrinal matter, from an empirical standpoint this coding scheme is purely arbitrary: it reflects the coder’s choice alone. It is impossible to view Oh’s study as contra-

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47 Oh, supra note 23, at 138.
48 Id. at 135.
49 See id. at 95 ("The versatility in the characterization of Fraud claims presents a potentially distortive effect on Thompson’s findings about the frequency of and rates for veil piercing in Contract and Tort.").
dicting or lending support to Thompson's because the former simply uses a different coding scheme to classify cases than the latter. The coder has determined the result, not the data.

Moreover, none of these studies evaluate any theory of veil piercing. It is true that Matheson finds statistically significant associations between certain case characteristics and the mechanically coded discussion of various topics with a veil-piercing outcome. But Matheson does not systematically demonstrate that one particular theory of veil piercing is a better predictor of courts' behavior than another. Rather, like Thompson, he shows mere associations between veil piercing and certain case characteristics and descriptive rationales. In this study, we supplement an in-depth qualitative discussion of several leading cases with automated content analysis, which uses statistical rules rather than arbitrary mechanical coding and systematically evaluates whether our theory is a better predictor of courts' behavior than traditional explanations.

Indeed, we believe that our taxonomy can produce a coherent account of veil-piercing cases, and are thus more optimistic than Stephen Bainbridge, who famously called for the abolishment of the doctrine. Unlike Bainbridge, we believe that there are strong public policy rationales for retaining veil piercing in certain situations. We hesitate to conclude that a century of jurisprudence represents a colossal mistake on the part of the courts in all fifty states. Rather, we suggest that three public policy rationales provide a systematic justification of veil piercing and that courts regularly decide in accordance with these rationales, even if they do not say so expressly.

II
A New Taxonomy

To say that the law is confused is not to say that the corporate veil should never be pierced. It is only to say that the corporate veil should never be pierced without a reason. We believe that there are three sensible reasons for piercing the corporate veil and that these justifications provide a complete account of the contexts in which courts find it appropriate to pierce the veil. As noted in the Introduction, these reasons are: (1) achieving the goals of a particular regulatory or statutory scheme; (2) avoiding fraud or misrepresentation by shareholders trying to obtain credit; and (3) promoting the bankruptcy value of eliminating favoritism among claimants to the cash flows of a firm.

50 See Matheson, supra note 46, at 4.
51 Bainbridge, supra note 15, at 479.
In the following sections, we discuss each of these justifications and present specific examples from cases found in the well-known treatise by Stephen Presser on piercing the corporate veil.\(^{52}\) In addition to its greater explanatory power than current theories, our framework offers two additional advantages over existing accounts of veil piercing. First, our three-part taxonomy distinguishes between situations in which courts actually pierce the corporate veil and situations in which the court does not ignore the legal form of a business organization but decides that certain liabilities that ostensibly are the corporation's alone actually are the exclusive or the shared responsibility of others, including shareholders and affiliate corporations. This latter category of cases, then, are not really piercing cases; rather, they are better characterized as joint and several liability cases. In particular, where a court finds liability on the basis that such liability promotes bankruptcy values, often the corporate veil of the defendant corporation is irrelevant: the relevant issue is the culpability of third parties, whose potential liability is in addition to the liability of the other entities.

Also, current accounts of the rationales for disregarding the corporate form, particularly accounts that consider undercapitalization as a grounds for veil piercing, fail to reconcile the basic tension between the well-settled doctrine that corporations can be established for the very purpose of avoiding personal liability and the doctrine that the veil can be pierced to avoid injustice. Under our account, corporations can be established for the sole purpose of avoiding personal or corporate liability on the part of investors but not when doing so is inconsistent with the goals of another regulatory or statutory scheme; when there is evidence of fraud or misrepresentation by companies or individuals trying to obtain credit; or when respecting the corporate form facilitates or enables favoritism among claimants to the cash flows of a firm.

Throughout this discussion, we omit details of the doctrinal differences between jurisdictions and focus on the primary rationales articulated by the courts in reaching the outcome. As we have already noted, the doctrinal terms in this area of law are so fluid and flexible that it makes little sense to distinguish between one court's reference to the "alter ego" theory and another court's emphasis on a "mere instrumentality." We are of the view that by examining the facts of the cases, the courts' holdings, and the primary rationales articulated by the judges within the doctrinal framework that applies, one can identify the underlying theories responsible for the outcome regardless of the terminology employed by the courts.

\(^{52}\) Stephen B. Presser, Piercing the Corporate Veil (2013).
A. Achieving the Purpose of a Regulatory or Statutory Scheme

1. Environmental Law

Perhaps the most straightforward theory that explains courts' decisions to pierce the corporate veil is furthering a regulatory or statutory scheme whose purpose would be undermined by upholding the corporate form. A classic example is the well-known Comprehensive Environmental Response, Compensation, and Liability Act of 1980 (CERCLA), which authorizes intervention by the Environmental Protection Agency in the wake of environmental disasters and imposes liability on those responsible for releasing hazardous substances into the environment.\(^53\)

Section 107(a)(3) of CERCLA imposes liability on "any person who by contract, agreement, or otherwise arranged for disposal or treatment . . . of hazardous substances."\(^54\) In *Carter-Jones Lumber Co. v. LTV Steel Co.*, the Sixth Circuit considered whether Harry Denune, the sole shareholder of defendant Dixie Distributing, could be held liable for Dixie's portion of the damages resulting from the release of hazardous polychlorinated biphenyls (PCBs) into the environment.\(^55\) After initially determining that veil piercing under section 9607(a)(3) was governed by Ohio state law,\(^56\) the Sixth Circuit imposed personal liability on Denune, expressly holding that the corporate veil may be pierced to further the goals of the CERCLA statutory scheme:

Congress enacted CERCLA with the intent of ensuring that those responsible for any damage, environmental harm, or injury from chemical poisons bear the costs of their actions. Courts applying state veil piercing law in conjunction with a CERCLA action must keep this statute's broad legislative purpose in mind. . . . Consider, for example, a case in which a corporation with a single shareholder kept immaculate corporate records, observed all the formalities required by corporate law, and was adequately capitalized. The shareholder never commingled funds, and never held himself out as personally liable for the corporation's debts. . . . Can it be that the shareholder is immunized from personal liability if he causes the corporation to commit an illegal act, no matter the degree of his control over the corporation with regard to the illegal act, no matter the harm to third parties, and no matter the other equities?\(^57\)

\(^{54}\) 42 U.S.C. § 9607(a)(3).
\(^{55}\) 237 F.3d 745 (6th Cir. 2001).
\(^{56}\) Carter-Jones Lumber Co. v. Dixie Distributing Co., 166 F.3d 840, 847 (6th Cir. 1999) ("If the corporate veil may be pierced under Ohio law to reach Denune as the sole shareholder, then he will be jointly and severally liable for Dixie's share of the response costs.").
This excerpt depicts the Sixth Circuit, correctly in our view, expressly disregarding various traditional elements of veil piercing—e.g., failing to observe corporate formalities and holding oneself out as personally liable for the corporations' debts—in order to further the goals of the CERCLA statutory scheme. Consistent with the taxonomy developed in this Article, the Sixth Circuit was unwilling to uphold limited liability if doing so would directly undermine a regulatory scheme that specifically rendered Denune's actions "illegal."

Similarly, in United States v. Kaysar-Roth Corp., the First Circuit affirmed a lower court holding that a parent corporation may be held liable for a subsidiary's violation of CERCLA when the parent is "involve[d] with 'operations having to do with the leakage or disposal of hazardous waste, or decisions about compliance with environmental regulations.""58 Kaysar-Roth involved a petition pursuant to Rule 60(b)(5) of relief from a 1990 declaratory judgment imposing liability for future cleanup costs on the parent corporation, Kaysar-Roth Corporation, for its subsidiary's release of trichloroethylene at a facility.59

In Kaysar-Roth, the First Circuit relied heavily on United States v. Bestfoods, where the Supreme Court emphasized that the mere exercise of shareholder control over a corporation through activities such as electing directors and approving bylaws is insufficient to render those shareholders personally liable for the corporation's violation of CERCLA.60 Indeed, even simultaneously serving as a director or officer of both the parent and subsidiary is insufficient to automatically attribute the actions of the latter to the former.61 Rather, "[t]he critical question is whether, in degree and detail, actions directed to the facility by an agent of the parent alone are eccentric under accepted norms of parental oversight of a subsidiary's facility."62 In this regard, direct involvement of the parent corporation in the subsidiary's compliance with environmental regulations would be sufficient to render the parent liable for the subsidiary's violation of CERCLA.63 In Kaysar-Roth, the First Circuit found that the parent corporation would remain liable because its executive vice president "played a central role in decisions about environmental compliance" at the subsidiary's facility.64

The Supreme Court's standard in United States v. Bestfoods reflects this first theory we have put forth for piercing the corporate veil,

59 Id. at 91.
60 Bestfoods, 524 U.S. at 61-62.
61 Id. at 69.
62 Id. at 72.
63 Id.
64 Kaysar-Roth, 272 F.3d at 104.
i.e., furthering the goals of a statutory or regulatory scheme. When a parent corporation becomes actively involved in a subsidiary's compliance with CERCLA, failing to pierce the veil would undercut the goal of the statutory regime by effectively granting the parent a partial exemption from the regulatory burdens the statute imposes. The parent would obtain the benefit of reducing the subsidiary's compliance costs—i.e., by shifting labor expenses from the subsidiary to the parent—while confining the risk of CERCLA liability to the subsidiary. Such "externalization" would give parent corporations an incentive to render subpar compliance by reducing the expected loss to the parent from CERCLA violations. The standard set forth in Bestfoods, demanding direct involvement of the parent in the subsidiary's compliance with environmental regulations, can thus be understood as directly furthering the goals of the CERCLA statutory scheme.

Environmental cases where courts refused to pierce the veil similarly comport with this rationale. In Raytheon Constructors Inc. v. Asarco Inc., the Tenth Circuit refused to impose personal liability on a shareholder corporation whose president also served as chairman and president of the subsidiary corporation.\(^{65}\) The court held that merely having an individual serve in this dual role did not lead to the automatic imposition of liability on the shareholder corporation. Indeed, the court emphasized that there was simply no evidence tying the shareholder corporation to the day-to-day operations of the subsidiary corporation's activities.\(^{66}\) The justification for veil piercing found in Bestfoods—ensuring that a parent cannot obtain the double benefit of partial regulatory compliance by shouldering the subsidiary's compliance costs while confining enforcement risk within the subsidiary—was simply not occurring in the facts of Raytheon Constructors. Accordingly, imposing liability on the shareholder corporation would not advance the goals of the CERCLA statutory scheme.

Similarly, in Browning-Ferris Industries of Illinois, Inc. v. Ter Maat, the plaintiffs sued two corporations in a contribution action under CERCLA for cleanup costs resulting from the defendants' improper abandonment of a landfill.\(^{67}\) The plaintiffs also sued Richard Ter Maat, the president and principal shareholder of the two corporations at the time. One of the issues before the Seventh Circuit was whether Ter Maat may be held liable personally for the cleanup costs under a veil-piercing theory. Following Bestfoods, the court concluded that were Ter Maat to have personally "supervised the day-to-day operations of the landfill... he would be deemed the operator, jointly with

\(^{65}\) 368 F.3d 1214, 1220 (10th Cir. 2003).

\(^{66}\) Id. at 1219–20.

\(^{67}\) 195 F.3d 953, 954 (7th Cir. 1999).
his companies, of the site itself" and thereby liable under CERCLA. The case was remanded because the district court failed to consider this question.

Indeed, the Seventh Circuit specifically rejected the application of other grounds for veil piercing other than the theories we articulate here. The court acknowledged that "the strongest case for piercing the veil is presented when the corporation had led potential creditors to believe that it was more solvent than it really was," which is the second rationale—deceiving creditors—that we discuss infra. In Browning-Ferris, of course, there was no such misrepresentation or induced reliance—the tort claimants were involuntary creditors forced to incur the cleanup costs pursuant to CERCLA. As a result, the court concluded, "[T]here is no issue of protecting reliance induced by misrepresentations by the debtor."

Moreover, the Browning-Ferris court rejected the claim the defendant corporations’ veil should be pierced because they were undercapitalized, emphasizing that "[t]he cases in which undercapitalization has figured in the decision to pierce the corporate veil are ones in which the corporation had so little money that it could not and did not actually operate its nominal business on its own." The court expressly concluded, as we argue throughout this Article, that "[u]ndercapitalization is rarely if ever the sole factor in a decision to pierce the corporate veil"; rather, it "is best regarded simply as a factor helpful in identifying a corporation as a pure shell, which [the defendant corporation] was not." Cases involving such a "pure shell" are encompassed by the second and third rationales we articulate infra, i.e., avoiding shareholder misrepresentation to creditors or promoting bankruptcy values. Indeed, as the Browning-Ferris court recognized, undercapitalization is an incoherent justification because no public policy purpose (e.g., encouraging efficient investment by diversified shareholders) is furthered by conditioning limited liability on a minimal level of capital.

2. ERISA

Section 406(b) of the Employee Retirement Income Security Act (ERISA) prohibits a fiduciary of a retirement plan from "receiv[ing] any consideration for his own personal account from any party dealing with such plan in connection with a transaction involving the as-

68 Id. at 956.
69 Id. at 961.
70 Id. at 959.
71 Id. at 960.
72 Id. at 961.
73 Id.
74 See id. at 960.
sets of the plan."\textsuperscript{75} This prohibition is intended to maximize the value of plan assets by ensuring objectivity, i.e., the distortion of professional judgment that may result from self-dealing or transactions involving conflicts of interest.\textsuperscript{76}

In a leading veil-piercing ERISA case, \textit{Lowen v. Tower Asset Management, Inc.}, the defendant-fiduciary Tower Asset Management invested plan assets in entities in which it held an equity interest, which facilitated a series of additional transactions that yielded a profit to the defendant.\textsuperscript{77} Such activity plainly violates the prohibition found in section 406(b) on a fiduciary receiving personal consideration for transactions involving plan assets. The Second Circuit held that in addition to Tower Asset Management, several of its shareholders may be held liable regardless of whether they were fiduciaries in their own right.\textsuperscript{78} The Second Circuit justified its holding by expressly discussing the need to uphold the purpose of the ERISA regulatory regime:

Neither the separate corporate status of the three corporations nor the general principle of limited shareholder liability afford protection where exacting obeisance to the corporate form is inconsistent with ERISA's remedial purposes. Parties may not use shell-game-like maneuvers to shift fiduciary obligations to one legal entity while channeling profits from self-dealing to a separate legal entity under their control.\textsuperscript{79}

The court continued:

The Supreme Court has 'consistently refused to give effect to the corporate form where it is interposed to defeat legislative policies.' . . . Courts have without difficulty disregarded form for substance where ERISA's effectiveness would otherwise be undermined. . . . \textit{A failure to disregard the corporate form in the circumstances of the present case would fatally undermine ERISA.}\textsuperscript{80}

In \textit{Lowen}, upholding the corporate form would have undermined ERISA because these shareholders likewise had an equity interest in the entities in which Tower Asset Management invested the plan assets in violation of section 406(b). Furthermore, Tower Asset Management "intermix[ed]" plan assets with these shareholders, rendering these individuals actively involved in Tower Asset Management's violation of section 406(b).\textsuperscript{81}

\textsuperscript{75} 29 U.S.C. § 1106(b) (2013).
\textsuperscript{76} See \textit{Lowen v. Tower Asset Mgmt., Inc.}, 829 F.2d 1209, 1213 (2d Cir. 1987) (discussing congressional concern with self-dealing by plan managers).
\textsuperscript{77} \textit{Id.} at 1214.
\textsuperscript{78} \textit{Id.} at 1220.
\textsuperscript{79} \textit{Id.} (emphasis added).
\textsuperscript{80} \textit{Id.} at 1220–21 (emphasis added).
\textsuperscript{81} See \textit{id.} at 1221.
It is essential to recognize that *Lowen* is a veil-piercing case because the Second Circuit did not hold that the shareholders were fiduciaries independently subject to section 406(b). Rather, liability was imposed because failing to do so would have given Tower Asset Management an incentive to engage in self-dealing as long as it ensured that the profits inured to a separate entity, i.e., its shareholders.\(^{82}\) Taken to the extreme, Tower Asset Management might be left with no assets if they were redirected in their entirety to the shareholders, rendering it impossible to collect damages on a suit for violating section 406(b) and eliminating any incentive to comply with the statutory ban on self-dealing. Accordingly, failing to pierce the corporate veil in *Lowen* would have undermined ERISA's goal of giving fiduciaries an incentive to make objective investment decisions in order to maximize the value of plan assets.

The *Lowen* court cited the First Circuit's decision in *Alman v. Danin*\(^{83}\) as an example of where "the incorporators of an inadequately capitalized corporation were held liable for the corporation's unpaid contributions to various pension plans."\(^{84}\) Nonetheless, a careful reading of *Alman* shows that undercapitalization was not the primary justification for the veil-piercing decision. The controversy in *Alman* involved the failure of a manufacturing company, Mohawk, to make its weekly contributions to its employee benefit plans.\(^{85}\) While the First Circuit briefly mentions that "[f]rom the outset, Mohawk was inadequately capitalized," this statement comes during a lengthy discussion of why piercing the corporate veil is justified:

> Indeed, deferring too readily to the corporate identity may run contrary to the explicit purposes of [ERISA]. . . . Congress enacted ERISA to ensure that employees were not deprived of promised benefits which they both expected and deserved. To permit [the individual defendants] to hide behind Mohawk, a paper corporation they knew was inadequately funded, and thereby avoid paying almost $100,000 in delinquent contributions would . . . tend to defeat ERISA's purposes and work a clear injustice.\(^{86}\)

This paragraph makes it clear that undercapitalization is only the means by which the individual defendants' failure to pay the employees led to a situation where upholding the corporate form would have defeated the statutory purpose of ERISA. In this specific setting, confining liability to the judgment-proof, bankrupt corporation would have left the employees without any recovery and thus without the

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82 See id. ("ERISA Section 406(b)'s prohibitions would be empty rhetoric if the corporate form might so easily shield those who profit from prohibited transactions.").

83 801 F.2d 1 (1st Cir. 1986).

84 *Lowen*, 829 F.2d at 1220.

85 *Alman*, 801 F.2d at 3.

86 Id. at 3-4.
employee benefits that ERISA was intended to ensure they would receive.

Indeed, in ERISA cases where the court refused to pierce the veil, this rationale was plainly lacking. In *Laborers' Pension Fund v. Litgen Concrete Cutting & Coring Co.*, the U.S. District Court for the Northern District of Illinois held that the failure of the defendant-corporation Litgen Concrete to make contributions under the terms of a collective bargaining agreement as required by section 145 of ERISA could not give rise to personal liability by William Litgen, one of its three shareholders who held a 40% equity interest in the firm. While the court mentions the traditional doctrinal grounds for piercing the veil—e.g., maintaining corporate records and adequate capitalization—its ultimate justification hinges on the statutory purpose rationale shown thus far:

The plaintiffs have submitted no facts demonstrating, for example, that William Litgen incorporated Litgen Concrete to defraud the plaintiffs or circumvent the law. The plaintiffs also have submitted no evidence that, were this court to hold Litgen Concrete liable, Litgen Concrete could not pay the judgment. Of course, were such to be the case, and the plaintiffs established the other prerequisite for piercing Litgen Concrete’s corporate veil, the plaintiffs could bring another action to pierce the veil and recover from Litgen Concrete.

While the *Litgen Concrete* court does not state its reasoning as eloquently as the First Circuit did in *Lowen*, the reference to Litgen Concrete’s ability to pay the judgment demonstrates that the underlying rationale of furthering the goals of the ERISA statutory scheme is dispositive. To be sure, we are not claiming that the mere fact that the corporation is unable to pay the judgment is sufficient to pierce the veil. Rather, the failure of a judgment-proof corporation to fulfill its ERISA obligations is often accompanied by some sort of diversion of corporate assets by the shareholders that renders it essential to disregard the corporate form to uphold the purpose of ERISA, i.e., to ensure that employees receive the benefits intended by the statutory scheme. In *Litgen Concrete*, the corporation’s solvency and the absence of any such asset diversion meant that veil piercing was unnecessary to ensure that the corporation fulfilled its collective bargaining contribution obligations under section 1145 of ERISA.

Similarly, in *Board of Trustees v. Valley Cabinet & Manufacturing Co.*, the Ninth Circuit refused to pierce the corporate veil in a suit

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89 *Id.* at 144 (emphasis added).
90 See, e.g., *Alman*, 801 F.2d at 4 (discussing how the defendant owners of a corporation "could not have expected [the corporation] to be able pay its debts").
91 877 F.2d 769 (9th Cir. 1989).
for unpaid collective bargaining contributions pursuant to section 451 of ERISA. At first glance, the facts in Valley Cabinet would seem to support veil piercing under the rationale we have articulated thus far: Valley Cabinet was a defunct, judgment-proof corporation, and the controlling shareholder of the corporation—Robert Davis—borrowed over $200,000 at the same time that the corporation failed to make these contribution payments. Indeed, these facts would seem well-suited to imposing liability on Davis under the theory that failure to pay the employee contributions would undermine the goal of ERISA's regulatory scheme.

In explaining its refusal to pierce the corporate veil, the Ninth Circuit noted the district court's finding that Davis lacked fraudulent intent when withdrawing these funds. Admittedly, at first glance, fraudulent intent may not seem relevant to furthering the purpose of ERISA's regulatory scheme. However, a careful reading of the opinion shows that the two are intertwined in this specific case:

There was evidence that Valley Cabinet did not believe that it owed money to the Fund. Valley Cabinet attempted to challenge the withdrawal liability assessment by requesting certain information from the Fund in 1982. The Fund failed to answer until it partially responded in 1985. Furthermore, Valley Cabinet's counsel advised Davis that Valley Cabinet did not owe contribution payments to the Fund, supporting an inference that Davis did not intend to defraud the trust when he transferred corporate assets to his personal account.

Conduct reflecting a bona fide dispute over whether the corporation actually owes the debt under ERISA does not undermine the goal of the regulatory scheme. A crucial fact in Valley Cabinet is that its collective bargaining agreement expired in June 1981, two months prior to the termination of contribution payments in August 1981. Moreover, in October 1981, the board of directors voted to close the business as of December 31, 1981. And when the employee fund in Valley Cabinet demanded over $200,000 in withdrawal liability, Valley Cabinet's corporate counsel informed the corporation's board of directors that it owed no money to the fund. Indeed, for at least ten years prior to this period, Valley Cabinet had been making its contribution payments pursuant to its collective bargaining agreements. Under these circumstances, it is difficult to conclude that ERISA's

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93 Valley Cabinet, 877 F.2d at 773–74.
94 Id. at 774.
95 Id. at 771.
96 Id.
97 Id. at 774.
98 Id. at 771.
purpose would be undermined by upholding the corporate form in
the wake of a bona fide dispute over Valley Cabinet's liability for non-
payment after the expiration of the collective bargaining agreement in
June 1981.

By way of contrast, in Lowen and Alman it was undisputed that the
shareholders' use of corporate assets by shareholders undermined the
ERISA regulatory regime by depriving employees of the benefits in-
tended by the statute. Indeed, there was no fraudulent intent in
Lowen, suggesting that the fraudulent intent theory emphasized by the
Ninth Circuit cannot serve as a universal justification for veil piercing.
The seemingly contradictory rationales emphasized in the aforemen-
tioned cases can be reconciled through the underlying rationale of
furthering the purpose of a regulatory scheme. When upholding the
corporate form implies undermining a statutory framework passed by
Congress to advance specific public policy goals, courts will intervene,
regardless of the doctrinal rhetoric they employ to justify such
intervention.

B. Avoiding Misrepresentation by Shareholders

A second justification for piercing the corporate veil is to prevent
the mistaken extension of credit, i.e., by shareholders who misrepre-
sent their active involvement in the subsidiary's affairs. In this section,
we will discuss this creditor-protection rationale and distinguish it
from the often-mentioned "undercapitalization" theory. It is true that
creditors are often harmed when a firm is insufficiently capitalized,
but in the contractual setting, this risk can be expressly allocated to
the corporations' principals by obtaining a personal guarantee of the
debt. Protecting creditors in the contractual setting thus turns on
preventing misrepresentations regarding shareholder involvement.
At the conclusion of this section, we will also discuss how this creditor-
protection rationale applies by analogy in the tort setting where the
corporation's failure to ensure sufficient oversight escalated the
probability of imposing harm on third parties.

1. Contract Creditors

In David v. Glemby Co., the plaintiff Jean-Louis David contracted
separately with defendants Glemby Co. and its 98%-owned subsidiary
Hair Programming, Inc. to license David's brand name and operate
Jean-Louis David hair salons in the United States.99 When HPI failed
to perform its contractual obligations, David brought suit against
Glemby as well, claiming that HPI's corporate veil should be pierced
because the two corporations acted as one entity in the transaction.100

100 Id. at 164-65. The breach of contract claims were against HPI alone. See id. ("HPI
The U.S. District Court for the Southern District of New York denied the defendants’ motion to dismiss on the veil-piercing claim, holding that a jury could reasonably find that HPI’s corporate form should be disregarded and liability imposed on Glemby.101 While the court mentioned the traditional elements of veil piercing—e.g., failure to completely observe corporate formalities—the court discussed these elements in the context of implied and express representations by Glemby that it would perform HPI’s duties under the contract:

Letters apparently written on behalf of HPI appeared on Glemby letterhead and referred ambiguously to “our obligation.” . . . David had no reason to believe HPI’s function was to avoid liability for Glemby, especially in light of David’s allegations that “Glemby promised Mr. David that it would take all steps necessary to see that Hair Programming performed the obligations stated in the Market Development Agreement, which Hair Programming signed, on instructions from Glemby.”102

It is apparent that the failure to observe corporate formalities that would permit the plaintiff to distinguish Glemby from HPI justifies disregarding HPI’s corporate form because Glemby implicitly misled the plaintiff into extending credit on the basis of Glemby’s involvement in HPI.103 Such misleading conduct does not rise to the level of common law fraud—indeed, there is no specific false statement— but justifies the imposition of liability because the extension of credit on the basis of a mistake regarding the involvement of the parent is inefficient.

A noteworthy case piercing the corporate veil in a setting similar to common law fraud is Mobridge Community Industries, Inc. v. Toure, Ltd.105 In Mobridge, the plaintiff MCI sued Toure and four of its directors for breach of contract for failing to make an installment payment for the sale of personal property.106 In imposing liability on the individual defendants, the Supreme Court of South Dakota emphasized the misrepresentation they made to MCI, which extended $250,000 in credit to Toure:

The Toure financial statement exhibited by the directors to MCI during the negotiations showed a net worth of $90,000, and a second statement showed a net worth of $65,000. This discrepancy was

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101 Id. at 167.
102 Id. at 167–68.
103 See id. ("David had no reason to believe HPI’s function was to avoid liability for Glemby.").
106 Id. at 130.
explained as the good will factor of approximately $25,000. At the
time the statements were exhibited to MCI, Toure had approxi-
mately $62.08 in its corporate account.107

_Mobridge_ is striking because the court held each of the four _directors_
liable under a veil-piercing theory, and there is no indication in the
opinion that these directors were also shareholders, although they
may have been.108 While it might seem most natural to bring a claim
against directors under a tort theory for fraudulent misrepresentation,
the _Mobridge_ court expressly imposed personal liability on the direc-
tors on the basis of the breach-of-contract claim against the
corporation.109

Like many piercing-the-veil cases, the _Mobridge_ court emphasized
the relative undercapitalization of Toure in light of the contractual
activity to be undertaken.110 But this case demonstrates precisely what
we have argued throughout this Article: rather than serving as an in-
dependent justification for piercing the veil, undercapitalization
serves to buttress one or more of the three primary rationales for dis-
regarding the corporate form that we have identified. In _Mobridge_, the
court imposed liability on the four directors because they misrepre-
sented Toure’s capitalization in order to obtain credit from MCI. The
court refers specifically to the directors’ “false representations” re-
garding their “financial ability to carry out the agreement.”111 Under-
capitalization in this setting, then, constituted the factual
representation that was falsely conveyed to the contractual
counterparty in exchange for an extension of credit. It did not serve
as an independent justification for the veil-piercing decision.

Moreover, the court specifically referred to the false misrepresen-
tations made by the directors with respect to their involvement in the
corporation’s activities, which weighed in favor of the decision to ex-
tend credit to the corporation:

The fact that [the individual defendants] were involved in the ven-
ture provided considerable credibility to the representations and
negotiations. Further misrepresentations came in the form of a
Toure management resume which included several reputable indi-
viduals in the business community who were to join Toure manage-

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107     _Id_. at 132.
108     _Id_. at 134 (“We conclude that the conduct of the directors constitutes the kind of
dealing which would make it inequitable for the trial court to recognize the corporate
entity of Toure and thus permit the individual directors to escape personal liability.”).
109     See _id_. (“The contract breached in this case was an obligation to pay money only
($250,000 in equal installments of $50,000, none of which was ever paid), and the trial
court finding that the amount due by the terms of the obligation was the proper measure
of damages cannot be held to be clearly erroneous.”).
110     See _id_. at 132 (“A further factor justifying a disregard of the corporate entity in this
case is inadequate capitalization.”).
111     _Id_. at 133.
ment within a month's time. There were other misrepresentations as to start-up time, worldwide patents owned or pending, availability of molds, plant improvements, and an influx of investment money.\footnote{Id.}

It is clear that the misrepresentation to obtain credit served as the primary justification for the court's decision and undercapitalization merely served as one of the factual premises at the center of such misrepresentation.

Indeed, the U.S. District Court for the Southern District of New York expressly reached a similar conclusion in Oriental Commercial and Shipping Co. v. Rosseel, N.V.\footnote{702 F. Supp. 1005 (S.D.N.Y. 1988).} The facts of Oriental are complex but essentially concerned a $34 million transaction between a Belgian oil trading corporation Rosseel and Oriental U.K., a U.K. corporation established and held by Abdul Hamed Bokhari, who also owned Oriental S.A., a Saudi Arabian corporation.\footnote{Id. at 1009.} Rosseel sought to hold Bokhari personally liable for Oriental U.K.'s alleged breach of contract, arguing that the corporate veil should be pierced because Oriental U.K. was extremely undercapitalized and represented that Bokhari and Oriental S.A. would stand behind the former's contractual obligations.\footnote{See id. at 1015–16 ("Rosseel is seeking to hold to the contract the principal on whose behalf its agent was apparently acting or to pierce the corporate veil.")}

Oriental is notable because the court expressly discussed the two theories we are contrasting in this section: undercapitalization per se as opposed to misrepresentation to creditors. While the court acknowledged that undercapitalization is an "element to be considered," it specifically held that "undercapitalization alone is not a sufficient ground for disregarding the corporate form" and "the mere fact that an entity may or may not have the capital to respond to a potential large award against it does not justify piercing the corporate veil."\footnote{Id. at 1020.} Indeed, the court emphasized that undercapitalization is "particularly important, where, as here, there has been a misrepresentation as to the assets of the company."\footnote{Id.} Just as in Mobridge, the undercapitalization in Oriental constituted the subject of the misrepresentation to creditors. The court's express emphasis on the inability of undercapitalization on its own to justify piercing the corporate veil is consistent with the argument we have advanced in this Article.
On the other hand, it is clear that the misrepresentation in Oriental did serve as a primary justification for the court’s decision to pierce the veil:

In sum, Bokhari represented to the world that his London office was part of his Saudi Arabian company. He hired an experienced oil trader known in the industry and permitted him to distribute the Saudia Arabian company’s promotional literature representing that the London company is part of the Saudi Arabian company. The representations were reasonably relied on and a large oil deal was entered into. After the deal collapsed, the Saudi Arabian company stated that the London office and Saudi Arabian company were “two different entities.” . . . Bokhari seeks the benefits of his representation that Oriental U.K. is a branch of Oriental S.A., but denies the liabilities. That is fraud and the Court will not allow the corporate form to be used for such a purpose.118

Indeed, this rationale expressly reflects the theory we have put forth in this section, namely, preventing shareholders from obtaining credit for the corporation by misrepresenting their personal involvement.

2. Tort Creditors

Unlike contract creditors, involuntary tort creditors present a more challenging setting for this rationale because there is generally no misrepresentation involved. It is in these cases that undercapitalization seems to arise as the sole explanation for piercing the veil and holding shareholders individually accountable for the corporation’s tort liability. However, a careful look at these situations shows that this generally only occurs when courts seek to discourage inadequate oversight or other forms of negligent management of the corporation. As such conduct can impose inefficient harms on third parties, courts’ intervention can be understood under a rationale analogous to preventing the “mistaken” extension of credit in a broader sense, i.e., the negligent infliction of involuntary harm.

A leading case often mistakenly cited for the proposition that undercapitalization alone is sufficient to pierce the veil is Minton v. Cavaney.119 In Minton, the Seminole Hot Springs Corporation operated a public swimming pool, and the plaintiffs obtained a judgment against Seminole for their daughter’s wrongful death by drowning in the pool.120 The plaintiffs sought to hold Cavaney, the director, secre-

118 Id. at 1023.
119 364 P.2d 473 (Cal. 1961). For an example of an appellate court incorrectly citing Minton for the proposition that undercapitalization is sufficient, see Nilsson v. Louisiana Hydrolec, 854 F.2d 1538, 1544 (9th Cir. 1988). Presser thoroughly rejects the notion that undercapitalization is sufficient to pierce the veil in California in PRESSER, supra note 52, § 2.5.
120 Minton, 364 P.2d at 474.
tary, and treasurer of Seminole, personally liable for the judgment. In imposing liability on Cavaney personally, the California Supreme Court emphasized Seminole's insufficient capitalization:

The equitable owners of a corporation . . . are personally liable when they . . . provide inadequate capitalization and actively participate in the conduct of corporate affairs. In the instant case the evidence is undisputed that there was no attempt to provide adequate capitalization. Seminole never had any substantial assets. It leased the pool that it operated, and the lease was forfeited for failure to pay the rent. Its capital was "trifling compared with the business to be done and the risks of loss." Despite the Minton court's rhetorical emphasis on undercapitalization, this was not the only deficiency in Seminole's conduct as a corporation. Specifically, the court emphasized that "section 800 of the Corporations Code provides that . . . the business and affairs of every corporation shall be controlled by, a board of not less than three directors . . . A person may not in this manner divorce the responsibilities of a director from the statutory duties and powers of that office." Seminole's failure to comply with an express statutory mandate provides a much more compelling basis to disregard the corporate form because it suggests inadequate oversight of the corporation. Inadequate oversight (i.e., by a board of directors that is smaller than the statutory minimum) can increase the probability that the corporation will act in a socially harmful manner. Were Seminole to have been subject to a larger board of directors and conducted its affairs in accordance with the statutory requirements, perhaps it would have taken greater care in preventing drowning in the swimming pool, i.e., by posting a lifeguard. The Minton court's emphasis on failure to observe formalities should be understood as reflecting a refusal to uphold limited liability when the corporation is operated in a manner that is significantly likely to impose harm on others and thereby render them involuntary tort creditors of the firm.

Indeed, as Judge Frank Easterbrook observed in Secon Service System, Inc. v. St. Joseph Bank & Trust Co.:

[1]t is a lot harder to hold investors personally liable in contract disputes than for tort judgments. The reason is simple: contract creditors have entered into a voluntary arrangement with the corporation, which gave them an opportunity to negotiate terms reflect-

121 Id.
122 Id. at 475 (citations omitted).
123 Id. at 475–76.
124 Cf. id. at 477 (Schauer, J., concurring and dissenting) ("[A] corporation cannot properly be regarded as organized and ready to even begin carrying on business until at least . . . directors and officers [are] elected.").
ing any enhanced risk to which doing business with an entity enjoying limited liability exposed them. If they wanted guarantees from the investors, they could have negotiated for them. Tort creditors had no chance to obtain compensation ex ante for exposure to increased risk, so to cut off all liability might encourage excessively risky behavior.\textsuperscript{125}

As support for his conclusion that veil piercing in the tort setting has a risk-reduction purpose, Easterbrook cited the Fifth Circuit’s decision in \textit{United States v. Jon-T Chemicals, Inc.}\textsuperscript{126} Jon-T Chemicals concerned a tort suit by the U.S. government for fraudulent misrepresentation and conversion against defendants John H. Thomas; Jon-T Farms, an Oklahoma corporation; and Jon-T Farms’ parent firm, Jon-T Chemicals.\textsuperscript{127} Jon-T Farms and Thomas were criminally convicted in a prior proceeding, and the government sought to hold Jon-T Chemicals liable under a veil-piercing theory for the judgment against its subsidiary, Jon-T Farms.\textsuperscript{128}

In explaining its decision to pierce the corporate veil, the Fifth Circuit emphasized that while “in contract cases, fraud is an essential element of an alter ego finding[,] . . . we do not require a finding of fraud in tort cases, particularly where the subsidiary is undercapitalized.”\textsuperscript{129} But as with \textit{Minton}, the court’s rhetorical emphasis on undercapitalization belies the true rationale for holding the parent responsible. The court’s justification makes it clear:

To mention just some of the evidence supporting the district court’s alter ego holding, all of the directors and officers of Farms served as directors and officers of Chemicals; Farms was wholly owned by Chemicals; Chemicals paid many of the bills, invoices, and expenses of Farms; it covered Farms’s overdrafts; it made substantial loans to Farms (at one time amounting to \$7 million) without corporate resolutions authorizing the loans and without demanding any collateral or interest; Chemicals and Farms filed consolidated financial statements and tax returns; Farms used the offices and computer of Chemicals without paying any rent; the salary of Farms’s one regular employee was paid by Chemicals; and employees of Chemicals performed services for Farms without charging for their time. Chemicals also advanced money and provided services on an informal basis to the joint ventures.\textsuperscript{130}

The emphasized text suggests that what was really motivating the court’s holding was the concern that the casual manner in which the parent-subsidiary relationship was managed increased the likelihood

\textsuperscript{125} 855 F.2d 406, 413–14 (7th Cir. 1988) (citations omitted).
\textsuperscript{126} 768 F.2d 686 (5th Cir. 1985).
\textsuperscript{127} \textit{See id.} at 688.
\textsuperscript{128} \textit{See id.}
\textsuperscript{129} \textit{Id.} at 692–93 (citations omitted).
\textsuperscript{130} \textit{Id.} at 695 (emphasis added).
that the subsidiary would impose inefficient harms on third parties. As with Minton, the judicial search for the "legitimacy" of the subsidiary's corporate existence finds its expression in specific actions of insufficient oversight that instrumentally justify piercing the corporate veil. The "alter ego" finding is not a metaphysical recognition of a unified parent-subsidiary identity but rather a rhetorical device that courts employ to justify discouraging management of the corporation that is likely to be socially harmful.

C. Bankruptcy Values

The goal of bankruptcy is to resolve the problems of insolvency in the most efficient way possible in order to preserve or dispose of the assets of the insolvent company or individual in the most efficient configuration possible, thereby maximizing the value of the firm for the creditors who are the residual claimants of companies that are insolvent. Among the most important factors in assuring a fair and efficient bankruptcy process are respecting the contractual and legal priorities of the various claimants and eschewing costly and duplicative individual creditor remedies in favor of a single coordinated process involving all of the various claimants to the insolvent firm's assets.

One of the primary ways that the doctrine of piercing the corporate veil is utilized is to protect against efforts to undermine the integrity of the bankruptcy process, particularly by controlling shareholders who may try various maneuvers on the eve of bankruptcy in order to benefit themselves at the expense of other claimants who are more deserving by dint of their contractual, common law, or statutory rights.

An example of bankruptcy values being furthered by the invocation of the doctrine of piercing the corporate veil is Stone v. Eacho. In Stone, the Delaware corporation Tip Top Tailors and its Virginia subsidiary by the same name were placed into bankruptcy proceedings upon becoming insolvent. The Delaware parent operated eight stores across the country, including the Virginia subsidiary, which was managed in a manner virtually identical to the other seven stores. The books and records of the Virginia subsidiary were maintained at the Delaware corporation, and the latter also handled the receipt of money into the subsidiary's treasury, the execution of contracts by the subsidiary, and the subsidiary's payment of wages to its employees.

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131 Jackson, supra note 5, at 17.
132 See id. at 16.
133 127 F.2d 284 (4th Cir. 1942).
134 Id. at 286.
135 Id.
In *Stone*, the Fourth Circuit pierced the corporate veil, holding that the assets of the parent should be available to fulfill claims by creditors of the subsidiary.\(^{136}\) But unlike the cases described in section II.B.1, there was no suggestion that either the parent or subsidiary engaged in any express or implied misrepresentation to creditors of either entity. Not only were the elements of common law fraud missing, there was simply no factual misrepresentation of the sort that would justify veil piercing to minimize the inefficient allocation of credit mistakenly extended as a result of a misrepresentation.

Rather, in *Stone*, the court pointed to a different rationale for piercing the corporate veil: furthering the efficiency and fairness of the bankruptcy proceeding.\(^{137}\) Once both the parent and subsidiary had reached the point of insolvency, it would be inefficient and inequitable to arbitrarily prefer creditors of the parent to those of the subsidiary by restricting each to the assets of the respective corporate entity. In the court’s own words:

> [W]here both corporations are insolvent, where the business has been transacted by and the credit extended to the parent corporation, and where the subsidiary has no real existence whatever, there is no reason why the courts should not face the realities of the situation and ignore the subsidiary for all purposes, allowing the creditors of both corporations to share equally in the pooled assets. . . . Perhaps the fairest way of dealing with the situation when both the parent and the subsidiary corporations are insolvent is to let all the creditors of each share pro rata in the pooled assets of both. Such procedure would be especially equitable where the claimants are creditors of both the parent and the subsidiary.\(^{138}\)

The fairness rationale to this holding seems relatively straightforward: there is no reason to arbitrarily prefer one set of creditors over another when both find themselves in the unfortunate position of collecting only cents on the dollar as a result of an unsuccessful extension of credit. The efficiency rationale, however, might seem less clear at first glance: one might wonder why a creditor of the subsidiary could not have simply demanded a guarantee by the parent had the creditor wished to satisfy a judgment from the parent’s assets in an insolvency proceeding.\(^{139}\) Why should the parent’s creditors “pay” for the subsidiary creditors’ failure to obtain an intrafirm guarantee? And subsidiaries could always obtain the opposite result by

\(^{136}\) Id. at 288.

\(^{137}\) See id. at 289 (“[I]n justice to all creditors, the corporate entity should be ignored.” (emphasis added)).

\(^{138}\) Id. at 288 (internal quotation marks omitted).

\(^{139}\) Cf. Secon Serv. Sys., Inc. v. St. Joseph Bank & Trust Co., 855 F.2d 406, 413–14 (7th Cir. 1988) (“If creditors wanted guarantees from the investors, they could have negotiated for them.”).
contract, suggesting that the only question is whether a default rule of consolidated liability is more optimal. But sophisticated, repeat contractors such as creditors are likely to be relatively unaffected by a default rule.

We suggest that where creditors of the subsidiary are aware of the parent’s coordination of the subsidiary’s business activities, it is highly likely that the extension of credit solely to the subsidiary reflects a mistake, the correction of which promotes an efficient outcome by more accurately reflecting creditors’ ex ante intent. To this end, the court in Stone noted:

There is nothing in the record before us to show that any of the creditors who filed claims in the bankruptcy proceedings of the Virginia corporation intended to extend credit particularly to that corporation; and the fact that the bills of the Richmond store were paid by the Delaware corporation from its Newark office would indicate that it must have been generally known to the creditors of the Richmond store that it was the Delaware corporation that was there engaged in business.\textsuperscript{140}

In addition, piercing the corporate veil in this setting promotes efficiency from the standpoint of the bankruptcy process. When both the parent and subsidiary have reached the point of insolvency, the legal system has an interest in reducing unnecessary, duplicative costs inherent in conducting parallel procedures to uphold the corporate identity of each of the subsidiaries. The Stone court expressly emphasized this rationale by concluding that “where the court decides that the corporate entity of the subsidiary should be completely ignored and its assets and liabilities treated as those of the parent corporation, it is both logical and convenient that this be done in one proceeding.”\textsuperscript{141} These substantive (credit-extending) and procedural rationales explain the tendency of courts to pierce the corporate veil in the bankruptcy setting.

In the years following Stone, bankruptcy law developed the doctrine of “substantive consolidation,” which pools the assets of separate debtors to satisfy creditor claims when “creditors dealt with the entities as a single economic unit and ‘did not rely on their separate identity in extending credit’” or “the affairs of the debtors are so entangled that consolidation will benefit all creditors.”\textsuperscript{142} In essence, then, substantive consolidation is simply an application of veil piercing that is animated by this rationale of protecting creditors and ob-

\textsuperscript{140} Stone, 127 F.2d at 287.

\textsuperscript{141} Id. at 289.

\textsuperscript{142} In re Augie/Restivo Baking Co., 860 F.2d 515, 518 (2d Cir. 1988) (citations omitted).
taining procedural benefits rather than preventing misrepresentation or furthering a statutory scheme.

A leading case demonstrating the limits of substantive consolidation—and thus veil piercing in the bankruptcy setting—is In re Augie/Restivo Baking Co.\textsuperscript{143} In Augie/Restivo, the Second Circuit refused to consolidate claims against two debtors who had exchanged stock as part of a contemplated merger, but one of the entities remained a shell corporation that held real property assets and continued to obtain credit despite the operating activities residing in the other entity.\textsuperscript{144} The Second Circuit emphasized that substantive consolidation should proceed from the default rule that creditors are capable of conditioning an extension of credit on obtaining cross-firm or interfirm guarantees:

\begin{quote}
[C]reditors who make loans on the basis of the financial status of a separate entity expect to be able to look to the assets of their particular borrower for satisfaction of that loan. Such lenders structure their loans according to their expectations regarding that borrower and do not anticipate either having the assets of a more sound company available in the case of insolvency or having the creditors of a less sound debtor compete for the borrower’s assets.\textsuperscript{145}
\end{quote}

However, the court emphasized, there was express evidence that some of the creditors knew they were dealing with separate entities.\textsuperscript{146} Moreover, in the absence of strong procedural justifications for consolidating the entities, the mere possibility that a consolidated proceeding would benefit creditors is insufficient:

Where . . . creditors . . . knowingly made loans to separate entities and no irremediable commingling of assets has occurred, a creditor cannot be made to sacrifice the priority of its claims against its debtor by fiat based on the bankruptcy court’s speculation that it knows the creditor’s interests better than does the creditor itself. The rationale of the bankruptcy judge in the instant case would allow consolidation of two completely unrelated companies upon a finding that the creditors would be better off under some proposed plan involving the joint sale of their assets.\textsuperscript{147}

\textsuperscript{143} \textit{Id.}
\textsuperscript{144} \textit{Id.} at 517.
\textsuperscript{145} \textit{Id.} at 518–19. It is worth noting that reasoning by reference to creditors’ “expectations” is circular because the law determines what those expectations are. Nonetheless, there are more analytically sound justifications for a default rule of limited liability such as encouraging portfolio diversification. \textit{See, e.g.}, Easterbrook & Fischel, supra note 16.
\textsuperscript{146} 860 F.2d at 519 (“It is undisputed that Union’s loans to Augie’s were based solely upon Augie’s financial condition, and that, at the time the loans were made, Union had no knowledge of the negotiations between Augie’s and Restivo. MHTC also operated on the assumption that it was dealing with separate entities. . . . Given these circumstances, the fact that the trade creditors may have believed that they were dealing with a single entity does not justify consolidation.”).
\textsuperscript{147} \textit{Id.} at 520.
Piercing the corporate veil in the bankruptcy setting through substantive consolidation necessitates demonstrating either a likelihood of a mistake on the part of a creditor in extending credit to only one of the entities or that consolidation would substantially promote efficiency in the bankruptcy process from a procedural standpoint.148

A recent example of the conditions for veil piercing being present in the bankruptcy setting is In re Brentwood Golf Club, LLC.149 In Brentwood, Barrie and Farrell Moore managed and held equity interests in two limited liability companies: a golf club, Brentwood Golf Club, and a tavern which operated at that golf club, Brentwood Tavern.150 The tavern enjoyed a below-market lease from the golf club, and Barrie Moore served as manager of both entities.151 Indeed, the two entities “did not keep organized or separate financial records,” and “Barrie Moore made the decisions, on a check by check basis,” as to which entity expense would be applied.152

When the golf club filed for bankruptcy protection, the trustee sought to pierce the corporate veil and substantively consolidate the assets of the tavern with the golf club to satisfy the latter’s creditors.153 The bankruptcy court granted the trustee’s request, emphasizing that “the two business [sic] are inextricably intertwined and all of the corporate formalities have been disregarded” because they share assets, employees, and creditors.154 It is notable that the court emphasized the twin aspects of the bankruptcy-values rationale that we have advocated in this section, preventing undermining of the bankruptcy process or directly furthering the aims of the process itself.155 As for the interfirm creditor mistake theory, the court concluded that the tight intertwining of the two entities led to “a presumption that creditors have not relied on the separate credit of each of the entities involved.”156

Moreover, from the standpoint of the bankruptcy proceeding, consolidation would create value for all creditors involved:

Tavern has failed to identify any specific creditor whose interest will not be served by consolidation. In fact, none of Tavern’s creditors, other than the Moores who are insiders, have objected to substantive consolidation. . . . In this case, substantive consolidation is

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148 See id. at 518.
150 Id. at 805.
151 Id. at 806.
152 Id. at 807.
153 Id. at 808.
154 Id. at 809.
155 See id. at 812 (“[T]he prima facie case [for substantive consolidation] is established upon: (1) a showing of 'substantial identity' . . . ; and (2) substantive consolidation is necessary to avoid a harm or produce a benefit for the estate.”).
156 Id. at 813.
appropriate . . . because "the affairs of the debtors are so entangled that consolidation will benefit all creditors."\textsuperscript{157}

In short, promoting bankruptcy values—the maximization of creditor value ex post from a procedural standpoint—is a third compelling rationale for piercing the corporate veil.

\textbf{III}

\textbf{EMPIRICAL STUDY}

In this Part, we test our proposed theoretical framework with an empirical study of 9,380 judicial opinions. We find that the three theories we articulate systematically predict veil-piercing outcomes; undercapitalization poorly explains these cases; and our three-part taxonomy most closely reflects the textual "structure" of veil-piercing cases, which, as we will explain, is the groups of words and phrases reflecting the underlying ideas found in the opinions. Before discussing our findings in detail, we first explain the unique empirical methodology we employ.\textsuperscript{158}

\textbf{A. Empirical Method: Text Analysis for Structure Discovery}

We employ a technique called \textit{text analysis}\textsuperscript{159} to test our theory. Text analysis measures the extent to which specific words

\textsuperscript{157} \textit{Id.} at 813–14.

\textsuperscript{158} In this Part, we refrain from using technical terms typically utilized in machine learning such as "training set," "loss functions," and so forth, but the reader who is familiar with these concepts will recognize their expression through simpler, more intuitive terminology.

\textsuperscript{159} Empirical studies of law and legal institutions typically use the method of \textit{causal inference}. While the precise design can vary from study to study, the basic approach is to articulate a theory of expected behavior in response to some intervention, such as a proposed regulation, and then to evaluate whether the intervention produces, on average, the expected outcome in an observed sample drawn from a larger population of interest. For example, one might evaluate whether informing consumers of empirically unexpected contract terms through a "warning box" leads to greater understanding of those terms as measured by a post-contracting quiz. See Joshua Mitts, \textit{How Much Mandatory Disclosure Is Effective?} (Working Paper, 2014), available at http://papers.ssrn.com/sol3/papers.cfm?abstract_id=2404526. If consumers exposed to the warning box have 10% more correct answers on average, one can conclude that the average benefit of the regulatory intervention is approximately 10%.

The fundamental requirement for a valid empirical study employing causal inference is \textit{conditional independence} with respect to the intervention of interest such as the regulatory policy. See Joshua D. Angrist & Jorn-Steffen Pischke, \textit{Mostly Harmless Econometrics: An Empiricist's Companion} 15 (2009). Put simply, conditional independence means that the intervention is \textit{independent of} every characteristic of units in the sample being studied, i.e., there is no systematic link between the intervention and some other attribute of the units in the sample. The requirement to achieve conditional independence poses a high hurdle for studies that merely observe interventions as they occur in the "real world"
(i.e., undercapitalization or fraud), or the absence of such words in a legal opinion or other document, increases the probability of an outcome occurring (i.e., a holding that the corporate veil should be pierced in a particular case). We do not mean to imply or to suggest that the mere presence of one specific word or phrase in a document is independently—regardless of any other content in the document—

because policies are generally distributed in society in a nonrandom manner, i.e., on the basis of some other characteristic of the individuals involved.

To take a simple example, government policies are often targeted to poor individuals, i.e., distributed on the basis of income. As a result, it is impossible to identify the effect of the policies merely by comparing outcomes (e.g., education, crime, etc.) between recipients and nonrecipients of the policies: the former group differs systematically from the latter by income. It is thus impossible to "disentangle" the effect of income and the effect of the policy, i.e., one cannot determine whether any differences are due to the policy, income, or both. This is also known as "selection bias" or "omitted variables bias": receipt of the policy is confounded by some other "omitted" characteristic which individuals are "selected" upon, i.e., income.

While multiple regression analysis can attempt to isolate the causal effect of an intervention by adjusting estimates by the effect of other characteristics such as income, multiple regression is inadequate when there are unobserved differences between the two groups at the basis of the comparison. Government policies that are available by choice, i.e., on an opt-in basis, predominantly feature this shortcoming: individuals who opt-in to receive the policy are likely to systematically differ from those who opt-out in many unobserved ways. For example, an individual's choice to attend college may reflect some inherent, unobserved drive or determination that those who do not attend college lack. Accordingly, it is impossible to attribute different outcomes (i.e., income levels) between college graduates and those lacking a college degree to higher education itself: the differences may instead be simply due to some unobserved difference between individuals in the population who choose to attend college and those who do not. Put differently, if there were some other way to measure this inherent drive or determination, one could "control" for its effect and obtain a better estimate of the true payoff of higher education. But since no such data are available, it is impossible to isolate the effect of a college education merely by observing different outcomes between the two groups.

The only certain remedy for "omitted variables bias" is to employ randomization. As Michael Abramowicz, Ian Ayres, and Yair Listokin have emphasized, random assignment of a policy to individuals in a population facilitates obtaining conditional independence on average. Michael Abramowicz, Ian Ayres & Yair Listokin, Randomizing Law, 159 U. Pa. L. Rev. 929, 935 (2011). One can attribute differences in outcomes between the "treatment" and "control" groups to the intervention because randomization ensures that assignment of the treatment will be uncorrelated with some other unobserved characteristic of units in the sample. Abramowicz, Ayres, and Listokin have called for policymakers to produce "randomization impact statements" to provide nonconfounded estimates of the payoffs of regulatory policy. Id. at 937.

However, despite the essential importance of causal inference to empirical research and legal policymaking more generally, not every empirical study necessarily evaluates a causal theory. Some theories do not attempt to make conditionally independent predictions of the average effect of an intervention in a population. Rather, they are inherently predictive in nature: they suggest that the presence or absence of certain factors increases the probability of a particular outcome, without claiming that those factors are necessarily "responsible" for the outcome in a conditionally independent, causal sense. Predictive findings can be important for policymaking because they describe what is presently working or not working, and indeed what is likely to work in the future when certain conditions are present, even though they cannot identify which factors are specifically responsible for the outcome.
responsible for increasing the probability of an outcome occurring.\textsuperscript{160} Rather this analysis assumes that words and phrases occur in joint combinations that reflect underlying ideas—rationales and justifications for an outcome expressed through coherent groups of related words and phrases.\textsuperscript{161}

We refer to these underlying ideas and the associated groups of related words and phrases as the "textual structure" of the judicial opinions. The goal of text analysis is to discover this structure, i.e., to identify the distinct ideas that are associated with an outcome and the words and phrases that constitute those ideas. To take an illustrative example, imagine that we applied text analysis to the following fictitious restaurant reviews in Table 1.\textsuperscript{162} To simplify, assume that each restaurant receives either a "good" or "bad" score (indicated above the fictitious review), and we wish to identify the ideas associated with a "good" restaurant and the related words and phrases most tightly associated with these ideas:

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One of the most common applications of predictive analysis is medicine: doctors frequently identify "risk factors" for certain diseases without being able to precisely identify the causal effect of each factor. It is known that high blood cholesterol, high blood pressure, diabetes, obesity, smoking, lack of physical activity, an unhealthy diet, and stress, \textit{jointly} increase the probability of coronary heart disease—the number one killer of adults in the United States, U.S. DEP’T OF HEALTH & HUMAN SERVS., NAT’L HEART, LUNG, & BLOOD INST., \textit{What Are Coronary Heart Disease Risk Factors?}, http://www.nhlbi.nih.gov/health/topics/topics/hd/ (last visited Sept. 20, 2014)—but it is impossible to say with certainty how much any one of these factors, conditionally independent of the others, increases the risk of heart disease. Nonetheless, these risk factors are featured predominately in public health campaigns and form the basis of medical advice given to millions of Americans. Empirical evidence showing a link between these factors and heart disease does support a theory that something about one’s lifestyle increases the likelihood of heart disease, even if it is impossible to disentangle the contribution of each of these predictors from the others.

\textsuperscript{160} The only exception might be words that are actually direct indicators of the outcome, such as "affirmed" or "denied," and even then a causal interpretation might prove problematic because these words generally do not \textit{always} indicate the outcome.

\textsuperscript{161} Even if one postulates that some underlying idea is consistently responsible for a particular outcome in a causal sense, one cannot directly measure that idea other than through the words through which it is expressed—and these words are inherently joint predictors of an outcome, just as risk factors for heart disease are joint predictors of a heart attack. For a more detailed discussion of the predictive analysis we employ as contrasted with traditional methods of causal inference, see the Appendix to this Article.

\textsuperscript{162} This example is loosely based on the "we8there" restaurant reviews found in Denis D. Maud & Fabio G. Cozman, \textit{Representing and Classifying User Reviews}, in ENIA ’09: VIII ENCONTRO NACIONAL DE INTELIGÊNCIA ARTIFICIAL, BRAZIL (2009).
In these examples, certain phrases are tightly associated with the restaurant score and others less so. For example, the phrase “food was awesome” only appears in reviews of good restaurants (1 and 4) and “service was terrible” only appears in reviews of bad restaurants (3 and 5). The phrase “nice atmosphere,” however, is associated with one good (2) and one bad (3) restaurant. At a simple level, text analysis permits distinguishing highly predictive phrases like “food was awesome” from those that are more ambiguous like “nice atmosphere.” We will show in this Part that the words and phrases associated with our three-part taxonomy are more akin to “food was awesome”—i.e., highly predictive of veil-piercing decisions—and undercapitalization is more akin to “nice atmosphere,” i.e., not a very strong predictor of judicial outcomes.

But text analysis permits more than identifying the predictive power of specific words and phrases. One could rightfully critique the foregoing example for merely showing that some words are relatively better than others, not that a certain set of words best describes the reasoning in veil-piercing opinions, as we have argued in the prior Part. However, text analysis also facilitates discovering the underlying ideas that give rise to groups of words and phrases, enabling a natural division of the text into conceptual categories such as the three-part taxonomy we proposed in the prior Part.

By “idea,” we mean a coherent notion that is a more meaningful and interpretable predictor of an outcome than isolated words and phrases. In this fictitious example, one can intuitively identify two underlying ideas associated with “good” or “bad” restaurants: food quality and service quality. A more complete analysis of the textual structure of these reviews would identify these two core ideas and the words and phrases that more specifically reflect those ideas in the text of the restaurant reviews.

A common method by which text analysis performs such “structure discovery” is based on co-occurrence, i.e., the simple assumption that words and phrases that occur together are likely to be related. In this fictitious example, the phrase “chef came to greet us personally” occurs with “food was awesome” in the review of
restaurant (1). One could conclude, then, that “chef came to greet us personally” is related conceptually to the same idea as “food was awesome”—i.e., food quality.\(^{163}\) However, this association is weak because the phrases only co-occur in one restaurant review. On the other hand, the phrase “ask several times for drinks” appears twice with “service was terrible,” suggesting that these two phrases are more strongly related and reflect the same underlying idea, i.e., service quality.

Textual structure, therefore, consists of two dimensions: predictive power with respect to veil-piercing decisions and strength of association as measured by co-occurrence across opinions. This can be represented visually as follows:

**FIGURE 1. VISUAL REPRESENTATION OF DOCUMENT STRUCTURE**

The goal of structure discovery is to identify the words and phrases found in the upper-right corner of this figure, i.e., those that are both most predictive of the veil-piercing outcome and tightly related to the same underlying idea(s). In this Part, we apply text analysis to show that our three-part taxonomy best describes the structure of veil-piercing opinions, i.e., the words and phrase found in the upper-right corner correspond more or less to the three rationales we discussed.

Indeed, we will also show that undercapitalization lies in the bottom-left corner: it is both weakly associated with veil-piercing outcomes and poorly linked to an underlying idea reflecting a coherent group of associated words and phrases. Crucially, we are not claiming that veil-piercing decisions do not refer to inadequate capitalization. Indeed, as the qualitative examples in Part II indicate, judges frequently offer undercapitalization as one among a litany of reasons to

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\(^{163}\) This association holds even though “chef came to greet us personally” has less predictive power than “food was awesome” because the former is associated with one bad restaurant (5) in addition to two good restaurants (1 and 2).
piece the veil. We will show, however, that notwithstanding the frequency with which this term is invoked, undercapitalization is an inferior way to understand veil-piercing opinions because it does not best reflect the textual structure of the reasoning found in these opinions.164

Our methodology improves on prior empirical studies of piercing the corporate veil not only by moving beyond merely counting the words and phrases that courts invoke in their opinions, but also by analyzing the internal structure of the entire text of these opinions using quantitative, objective statistical techniques. This analysis reveals that the internal structure of a large sample of cases is consistent with our three-justifications-for-veil-piercing hypothesis.165 Significantly, our approach is more replicable and objective than prior empirical studies on veil piercing because we analyze the full text of the opinions rather than employ subjective coding of case characteristics.

The design of our study consists of four stages, which we summarize visually as follows:

![Figure 2. Empirical Design](image)

We explain each of these phases and present results in the following sections.

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164 Indeed, the goal of our empirical project is not to determine simply which words and phrases predict judicial outcomes—and certainly not to claim that any specific word, phrase, or group of the foregoing has a causal (i.e., conditionally independent) effect on the outcome—but rather that the predictive power of words and phrases reveals a natural, "latent" structure that is consistent with our three-justifications-for-veil-piercing hypothesis. By "latent" structure, we mean a taxonomy that is not immediately evident upon observing a particular case, but one that is revealed by examining the mathematical and statistical properties of the text of these opinions taken together.

165 We are not aware of any other study that has employed the entirety of the textual analysis methods we utilize here, although an article has applied a subset of the techniques that we employ here. Daniel Young uses algorithmic topic modeling to analyze 19,000 pages of newspaper text from the nineteenth century and finds empirical support for Bruce Ackerman's argument that constitutional change outside the amendment process can occur when there is significant popular attention to constitutional questions. Daniel Taylor Young, Note, How Do You Measure a Constitutional Moment? Using Algorithmic Topic Modeling to Evaluate Bruce Ackerman's Theory of Constitutional Change, 122 YALE L.J. 1990 (2013).
B. Classifying the Random Sample

We obtained 9,380 judicial opinions by searching across federal and state cases referring to either piercing the corporate veil or disregarding the corporate form.\footnote{As the mass downloading and retention of cases violates the Westlaw and Lexis license agreements, we utilized the free service provided by the CourtListener API, which contains the full text of approximately 2.5 million judicial opinions in the federal and state courts. \textit{Non-Profit Free Legal Search Engine and Alert System, Court Listener, https://www.courtlaw.com} (last visited Sept. 21, 2014). The search terms we utilized were "pierc* veil"-3 OR "disregard* corpor*"-5", where the syntax "-x" is similar to the "w/" syntax in Westlaw.} As a first step, we classify a random sample of 1,000 cases into three groups: (1) cases where the court pierced the corporate veil; (2) cases where the court refused to pierce the corporate veil; and (3) cases that contain language commonly found in piercing-the-corporate-veil or disregarding-the-corporate-form cases but do not actually apply this remedy.\footnote{This is a significant group because many of the cases containing these terms simply refer to the doctrines in passing without evaluating the facts in any way, or otherwise contain spurious references to these words in other contexts. By way of example, courts often referred to the notion of "piercing" the "veil" of a petition lacking substantive claims.}

As we noted previously, prior empirical studies on veil piercing have utilized mechanical coding of case characteristics, i.e., whether a judge utilized a particular rationale such as fraud or undercapitalization, in addition to coding case outcomes.\footnote{See, e.g., Matheson, supra note 46; Oh, supra note 23.} There is a crucial difference between the two, however: outcomes are often objective, certain, and verifiable (i.e., whether a court pierced the veil in a particular case). Case characteristics are much more subjective, as it is generally impossible to determine whether a decision merely mentioned and discussed a particular rationale or whether the court actually based its decision on it. Moreover, reducing the complexity of the textual content of a judicial decision to a limited sequence of case characteristics permits only a very "rough" hypothesis test, whereby the validity of one's argument depends entirely on whether the subjective coding scheme is accurate. For these reasons and others, mechanical coding of covariates is discouraged in the social sciences in favor of direct measurement of quantities of interest.

Admittedly, even classifying cases into these three categories of outcomes presented difficulties. For example, when denying an attempt to pierce the veil, courts often emphasize the absence of evidence sufficient to establish a veil-piercing claim, which leads to an asymmetry in the textual content of decisions in groups (1) and (2): denials are often summary and brief whereas veil-piercing findings consist of lengthy applications of the doctrine to the facts of the specific cases. Such asymmetry increases the amount of noise present in the data, but it is not particularly problematic because we find unique
factual content among cases denying veil-piercing claims in sufficient quantity to permit us to distinguish between the two groups.

That said, we adopted a fairly generous approach to including denials. We coded any case as a veil-piercing holding even if the court articulated several rationales for its rejection of a plaintiff’s motion, as long as one of these rationales consisted of an application—however brief—of some veil-piercing legal standard to the facts. We treated denials of motions to dismiss a veil-piercing claim as veil-piercing holdings because a denial of a motion to dismiss is a legal determination that the facts, as pleaded in the complaint, are sufficient to state a veil-piercing claim. Similarly, appeals vacating lower court veil-piercing holdings for lack of evidence were coded as veil-piercing denials because it reflects the opinion of the court that the evidence was legally insufficient to state a claim of veil piercing.

Our methodology was as follows. A case was coded as 1 if the court found at least one ground to pierce the corporate veil, -1 if no grounds were obtained, and 0 if the case merely mentioned the doctrine without deciding the case upon it. The key requirement for a 1 or -1 coding is that the court applied the doctrine to the facts (whether proven or merely pled) and rendered a legal conclusion. The failure to plead a veil-piercing claim was coded as 0 unless the court expressed an opinion on the viability of a veil-piercing claim on its own initiative. Often, plaintiffs sought to exercise personal jurisdiction over a shareholder under a veil-piercing theory. The legal standard for exercising personal jurisdiction often varied from the legal standard for imposing liability with the latter employing the traditional doctrinal tests but the former emphasizing constitutional due process considerations. Cases that were rejected from a jurisdictional standpoint purely on due process or other constitutional or statutory grounds were coded as 0 unless the court analyzed the jurisdictional claim under one or more substantive liability standards.

We recognize that no coding scheme perfectly captures the underlying complexities presented by judicial opinions, but with respect to the specific task of classifying opinions as veil-piercing grants/denials or irrelevant, our coding scheme is relatively objective and thus likely to lead to little variation between coders. No other aspect of our project depends on mechanical coding.

C. Predicting the Dataset and Verifying Accuracy

We assemble a random sample of piercing cases so that we can apply predictive coding classifications to the remaining cases in our dataset. While our use of predictive coding will inevitably contain some inaccuracies, mechanically coding every case in the dataset would be cost-prohibitive in light of its large size. Moreover substitut-
FINDING ORDER IN THE MORASS

ing human coding for predictive coding would introduce even greater inaccuracy due to its subjective nature because it is impossible to identify or measure subconscious deviations between the accuracy of mechanical coding on a large dataset.

We utilize the common predictive coding algorithm for text classification known as the naive Bayes classifier. The naive Bayes algorithm reflects a probabilistic approach to classification whereby a probability is estimated for each document being of each type (i.e., a veil-piercing decision, a denial of a veil-piercing claim, or an irrelevant decision), and each document is classified into the type with the highest probability. The naive Bayes algorithm makes a further simplifying assumption that the predictors are conditionally independent of each other.

In algebraic terms, the naive Bayes classification decision rule is given by: \( \arg\max_c \Pr(X \mid Y = c) = \prod_{j=1}^{d} \Pr(X_j \mid Y = c) \) where \( X_j \) is the \( j \)th predictor, i.e., word or phrase in a judicial opinion, \( X \) is the set of all predictors for that opinion, \( Y \) is the opinion's (unknown) true type, and \( c \) is the opinion's classified type, which is chosen so as to maximize the probability of all the predictors in that opinion conditional on that document being of type \( c \).

It is worth discussing how the \( X \) predictors are obtained for each opinion. Following customary conventions for text analysis, we employ the standard content-centric approach, colloquially known as the "bag of words" assumption, which partially disregards word order to obtain greater precision in measuring document content. As one of us has explained, the bag of words assumption means that:

[E]ach word in a document is viewed as pulled out of a hat, such that presence of prior words has no effect on the probability of any future word being pulled out. This means that the content of a document can be inferred solely from the frequency of word occurrence . . . . The bag-of-words assumption is plainly inaccurate, as language contains significant lexical structure. The existence of grammatical rules essentially means that some words are more likely

\[169\] See Trevor Hastie et al., The Elements of Statistical Learning: Data Mining, Inference, and Prediction 210 (2d ed. 2009); Max Kuhn & Kjell Johnson, Applied Predictive Modeling 353 (2013).

\[170\] While this conditional independence assumption is plainly incorrect, an extensive body of literature has shown that the naive Bayes classifier is surprisingly accurate for text classification, likely because the covariance between words and phrases is largely negligible across a vocabulary (even if non-negligible for certain words and phrases). See, e.g., David D. Lewis, Naive (Bayes) at Forty: The Independence Assumption in Information Retrieval, in Machine Learning: ECML-98, at 4 (Claire Nédellec & Céline Rouveiro eds., 1998); Harry Zhang, The Optimality of Naive Bayes, PROCEEDINGS OF THE SEVENTEENTH INTERNATIONAL FLORIDA ARTIFICIAL INTELLIGENCE RESEARCH SOCIETY CONFERENCE 562 (Valerie Barr & Zdravko Markov eds., 2004).

\[171\] See Daniel Jurafsky & James H. Martin, Speech and Language Processing § 19.2 (2008). More specifically, we utilize a "bag of bigrams" model whereby we treat the order of two-word phrases as ignorable.
than others to follow any given word. The inaccuracy of the bag-of-
words assumption does necessarily mean it lacks utility. In light of
the vast size of the English vocabulary, correlations between any two
individual words . . . are likely to be extremely weak. It is simply not
the case that two words often invariably follow each other unless
they form a common two-word phrase, which is relatively infre-
quent. Accordingly, these correlations can be treated as largely
ignorable, i.e., they should not affect inferences as to the content of
any given document based solely on the presence of individual
words.\textsuperscript{172}

Because we examine two-word phrases rather than single words, the
bag of words assumption is even more plausible, because it is even less
likely that two sets of two-word phrases will systematically follow each
other.

To generate the $X$ predictors, we preprocess the text as is com-
mon in text analysis\textsuperscript{173} and "tokenize" the opinions by dividing the
text into one-, two-, and three-word phrases. To understand how the


\textsuperscript{173} We preprocess our corpus of judicial opinions by converting all text to lowercase, removing numbers and punctuation, stemming each opinion using the standard Porter stemmer, M.F. Porter, An Algorithm for Suffix Stripping, reprinted in KAREN SPARCK JONES & PETER WILLET, READINGS IN INFORMATION RETRIEVAL (1997), removing a list of "stop words" that lack substantive meaning, and stripping extraneous white space from the document. These are standard preprocessing steps in automated content analysis to ensure that words reflect underlying content and not merely idiosyncratic prefixes or suffixes. See Justin Grimmer & Brandon M. Stewart, Text as Data: The Promise and Pitfalls of Automatic Content Analysis Methods for Political Texts, 21 Pol. Analysis 267 (2013).

\textsuperscript{144} [Vol. 100:99]
two-word phrases are generated, consider the sentence "We refuse to pierce the corporate veil." This sentence would be transformed ("tokenized") into the following two-word phrases:

<table>
<thead>
<tr>
<th>Table 2. Example of Two-Word Phrase Transformation</th>
</tr>
</thead>
<tbody>
<tr>
<td>- We refuse</td>
</tr>
<tr>
<td>- refuse to</td>
</tr>
<tr>
<td>- to pierce</td>
</tr>
<tr>
<td>- pierce the</td>
</tr>
<tr>
<td>- the corporate</td>
</tr>
<tr>
<td>- corporate veil</td>
</tr>
</tbody>
</table>

We reduce unnecessary dimensionality by removing so-called "sparse" terms that appear in fewer than 1% and 2% of the opinions, yielding lists of 7,435 and 2,171 two-word phrases, respectively. These two vocabularies are utilized for different portions of the analysis. The naive Bayes algorithm is sensitive to over-fitting and performs better with fewer phrases where each has greater predictive power. Accordingly, the 2,171-phrase vocabulary is used to classify the unlabeled opinions, but the broader vocabulary list of 7,435 phrases is used for the substantive analysis in subparts III.D and III.E infra.

These preprocessing steps allow us to create a document-term matrix, in which each row in the matrix represents a discrete judicial opinion, while the columns represent the two-word phrases. Each cell value is the number of times the specified two-word phrase appears in the opinion. The co-occurrence of each two-word phrase in the opinions belonging to class \( c \) empirically estimates the conditional distribution. We apply the naive Bayes classifier to the remaining 8,380 unclassified opinions in the dataset to obtain predicted types for each judicial opinion, i.e., a veil-piercing holding, rejection of a veil-piercing claim, and an irrelevant decision.

While it is impossible to compute the accuracy rate of the naive Bayes classifier without mechanically coding each of the remaining cases in the dataset, applying appropriate statistical methodologies enables us to estimate the accuracy rate of the naive Bayes classifier for the entire dataset. The specific technique we utilize is called \( k \)-fold cross-validation.\(^{175}\) Cross-validation is a resampling method whereby the coded random sample is divided into a number of groups—known as "folds"—generally five or ten. A predictive model is estimated on all but one of the groups (i.e., four of the five or nine of the

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\(^{174}\) Eliminating sparse terms is a common step in automated content analysis and is beneficial both for computational and substantive reasons. The frequency of word occurrences tends to follow the so-called "Zipf's" law for rare events, i.e., the vast majority of words appear in a very small number of documents. See generally M.E.J. Newman, Power Laws, Pareto Distributions and Zipf's Law, 46 CONTEMP. PHYSICS 323, 327 (2005). These words have very little predictive power (since they do not appear across many documents) and impose a substantial computational burden.

\(^{175}\) See GARETH JAMES ET AL., AN INTRODUCTION TO STATISTICAL LEARNING: WITH APPLICATIONS IN R 176 (2013).
ten) and then applied to the remaining group to obtain an accuracy rate for that group. The process is repeated for each of the groups and the group accuracy rates are averaged to obtain an overall accuracy rate obtained by cross-validation.\footnote{Id. at 176–86.}

Statistical theory suggests that the cross-validation accuracy rate generally will approximate the accuracy rate of the predictive model for the entire dataset, although it may be somewhat optimistic.\footnote{Id. at 182–83.} The basic intuition behind cross-validation is that resampling the population along five or ten groups and evaluating a model's performance on each subset permits making accurate inferences with respect to the predictive performance of a model when applied to the population by balancing the bias-variance tradeoff. Dividing the data into the groups "forces" the model to adapt to the inherent variation present in the data, which is similar to estimating the model on the observations in the population lacking a known class.

We perform ten-fold cross-validation and average the accuracy rate across the random sample that has been labeled with the type of opinion. To get a sense for how well the classifier performs, imagine if we had randomly categorized each opinion into one of the three groups, i.e., a veil-piercing holding, rejection of a veil-piercing claim, and an irrelevant decision. Statistical theory suggests that choosing one of the three categories at random in a sufficiently large sample should lead to an accurate classification approximately 33% of the time. Table 3 summarizes accuracy rates from one instance of ten-fold cross-validation using our classifier along with a comparison of merely choosing a type of opinion (-1, 0, or 1) at random for each of these folds.

<table>
<thead>
<tr>
<th>Fold</th>
<th>Naive Bayes Classifier</th>
<th>Random Classification</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>75%</td>
<td>36%</td>
</tr>
<tr>
<td>2</td>
<td>74%</td>
<td>32%</td>
</tr>
<tr>
<td>3</td>
<td>83%</td>
<td>29%</td>
</tr>
<tr>
<td>4</td>
<td>76%</td>
<td>44%</td>
</tr>
<tr>
<td>5</td>
<td>75%</td>
<td>38%</td>
</tr>
<tr>
<td>6</td>
<td>66%</td>
<td>27%</td>
</tr>
<tr>
<td>7</td>
<td>78%</td>
<td>30%</td>
</tr>
<tr>
<td>8</td>
<td>80%</td>
<td>44%</td>
</tr>
<tr>
<td>9</td>
<td>75%</td>
<td>37%</td>
</tr>
<tr>
<td>10</td>
<td>84%</td>
<td>35%</td>
</tr>
<tr>
<td>Average</td>
<td>77%</td>
<td>35%</td>
</tr>
</tbody>
</table>
As the table shows, the accuracy rates of our classifier are significantly higher than the expected 33% accuracy rate when the type of opinion is chosen at random. To estimate the variance of the accuracy rate estimates, we perform the ten-fold cross-validation ten times, and obtain an average accuracy rate of 76.62% with a standard deviation of 1.20%. These results indicate that the accuracy of our predictive model is consistently high compared to random classification.

D. Regressing Case Outcomes on the Text of the Opinions

After obtaining predicted outcomes for the remaining 8,380 opinions, we proceed to identify which two-word phrases have the greatest correlation with each of the two veil-piercing outcomes, i.e., denial or granting of a veil-piercing claim, which are coded as -1 or 1 in our dataset. Performing this step enables us to determine the particular phrases that predict veil-piercing decisions, and as such, we restrict our dataset to those opinions in which the court actually ruled on the veil-piercing issue. To identify these phrases, we utilize multinomial inverse regression (MNIR), a form of high-dimension regression analysis specifically designed for textual data represented by the bag-of-words form of phrase counts per document.\(^1^{78}\)

We apply MNIR to the subset of the opinions coded -1 or 1, which number 5,475 in total, to identify which of the 7,435 two-word phrases in the larger vocabulary best predict judicial decisions to pierce or refuse to pierce the corporate veil.\(^1^{79}\) MNIR produces a factor loading “coefficient” \(\hat{o}_j\) for each phrase in the vocabulary, which represents that phrase’s correlation with the dichotomous outcomes \(y\), as denoted by our model (see Appendix). In the following section, we apply a systematic method of grouping these phrases to render the results more interpretable, but we first present suggestive evidence of the superiority of our three-part taxonomy.

As a heuristic demonstration, we group the phrases into four categories, one for each of our three justifications for veil piercing discussed above, with the fourth reserved for the piercing justification that a corporation was undercapitalized. As we noted previously, we filtered out conclusory phrases reflecting the veil-piercing doctrine itself (e.g., “alter ego”) because they provide no insight into the judicial reasoning behind veil-piercing decisions. As the purpose of this analysis is to identify which substantive, meaningful phrases explain veil-piercing outcomes, these conclusory terms merely constitute “noise” because they shed no light on which substantive rationales are actually

\(^{178}\) For a technical overview of MNIR, see the Appendix to this Article.

\(^{179}\) Readers familiar with regression analysis will recognize that it would be difficult to fit a logistic regression model to this data by maximum likelihood because the number of predictors is so relatively high.
driving judicial decisions. We also eliminated random grammatical phrases that happen to correlate with the outcome variable and, for this heuristic demonstration, we ignore other terms that cannot be classified into one of these four categories. (We take a more robust approach in the following section.) The following table shows the top ten phrases that predict veil-piercing decisions for each of the categories along with the magnitude of their MNIR coefficients, which reflects the strength of the correlation.\(^\text{180}\)

**Table 4. MNIR Top Ten Phrases by Category**

<table>
<thead>
<tr>
<th>Statutory Scheme</th>
<th>Misrepresentation</th>
<th>Bankruptcy Values</th>
<th>Undercapitalization</th>
</tr>
</thead>
<tbody>
<tr>
<td>workmen['s] compen[sation] (1.9181)</td>
<td>actual fraudul[ent] (1.4812)</td>
<td>trust[e] power (2.1574)</td>
<td>invest[ment] capac[it]alization</td>
</tr>
<tr>
<td>worker['s] compens[ation] (1.5429)</td>
<td>scheme defraud (1.3202)</td>
<td>author bankrupt[cy] (1.8771)</td>
<td>amount capit[alization]</td>
</tr>
<tr>
<td>hazard[ous] substance[e] (1.4838)</td>
<td>individu[al] creditor (1.2299)</td>
<td>chapter liquid (1.7083)</td>
<td>asset satisfi[ed]</td>
</tr>
<tr>
<td>releas[e] hazard[ous] (1.3618)</td>
<td>mislead[ing] statement (1.2208)</td>
<td>debtor possess[ion] (1.6653)</td>
<td>capit[al] asset</td>
</tr>
<tr>
<td>hazard wast[e] (1.3194)</td>
<td>fraudul[ent] transfer (1.1229)</td>
<td>trust[e] avoid (1.5112)</td>
<td>gross</td>
</tr>
<tr>
<td>cerc la usc (1.2566)</td>
<td>neglig[ent] misrepresent[ation] (0.7960)</td>
<td>debtor incur[red] (1.3972)</td>
<td>capit[alization]</td>
</tr>
<tr>
<td>statut[ory] scheme (0.8562)</td>
<td>misrepresent[ation] (0.7878)</td>
<td>code bankrupt[cy] (1.3729)</td>
<td>inadequ[ate] capit[alization]</td>
</tr>
<tr>
<td>act cerc la (0.8165)</td>
<td>fals[e] mislead[ing] (0.7296)</td>
<td>accord bankrupt[cy] (1.3397)</td>
<td>capit[al] contribut[ion]</td>
</tr>
<tr>
<td>statut[ory] constr[u]cion (0.8070)</td>
<td>inten[tional] misrepresent[ation] (0.5635)</td>
<td>bankrupt[cy] trust[e] (1.2237)</td>
<td>only[y] asset</td>
</tr>
<tr>
<td>Average: 1.3194</td>
<td>Average: 1.0696</td>
<td>Average: 1.6244</td>
<td>Average: 0.4435</td>
</tr>
</tbody>
</table>

The data reflected in the table show that the coefficients for undercapitalization are significantly lower than for the other three theories. In fact, the phrases that unambiguously refer to undercapitalization—e.g., "inadequ[ate] capit[alization]" and "gross undercapit[alization]"—are among the least likely to be systematically

\(^{180}\) While it might seem counterintuitive at first, we present only the absolute value of MNIR coefficients because of the asymmetric nature of legal doctrine and judicial reasoning. When courts refuse to pierce the veil, they articulate the elements of the veil-piercing doctrine and emphasize what is missing from the facts, i.e., the facts which would have rendered the doctrine applicable. Similarly, when courts pierce the veil, they state which facts weigh in favor of the decision. Accordingly, the strongest textual predictors of veil-piercing outcomes are the dispositive facts that courts emphasize, regardless of the outcome of the opinions. The weakest predictors are terms that are "random," i.e., equally likely to be associated with either outcome.
associated with either outcome. We listed all of the phrase coefficients in an online appendix to this Article to demonstrate that we have not selectively ignored other phrases that may be plausibly interpreted as referring to undercapitalization.181

MNIR facilitates dimension reduction in a predictive sense, not causal inference: the factor loading coefficients cannot be interpreted as anything approaching a causal effect for each phrase but rather the degree to which each phrase "absorbs" the underlying variation in case outcomes in one direction or another. Indeed, there is undoubtedly substantial collinearity between the phrases, as a paragraph or sentence by definition consists of several related words and phrases. More fundamentally, as we have emphasized, a veil-piercing theory will necessarily be expressed through groups of related words that reflect the theory's underlying idea. As such, any attempt to model veil-piercing decisions causally—i.e., to identify independent theories—must operate on the level of these ideas that give rise to the observed text, as we discuss in the following section.

E. Discovering Theories Through Topic Modeling

The top-ten lists in the prior section represent a heuristic approximation of latent modeling. We simply grouped the two-word phrases by their intuitive and logical link to the theory we put forward in this Article.

A great deal of research has been dedicated to discovering latent structure in text, which is the process of identifying the underlying ideas that give rise to the words and phrases that we observe in documents such as judicial opinions. In this section, we apply the technique known as topic modeling to discover the textual structure in the judicial opinions in our dataset of piercing-the-corporate-veil cases at the level of coherent ideas.182

Topic modeling is essentially a "mapping" of particular words or phrases to a much smaller set of distinct ideas or particular concepts. Mapping is probabilistic. Each document is viewed as a weighted "mixture" of topics. Each of these topics consists of a probabilistic mapping of words or phrases to that topic. For example, the three phrases "to pierce," "pierce the," and "corporate veil" might be associated with one topic with probabilities 1%, 2%, and 3%, respectively. Docu-


182 Topic modeling has been used to test the validity of a constitutional theory. See Young, supra note 165 (applying topic modeling to historical newspapers to support a constitutional law argument); see also Mitts, supra note 172, at *27, *40 (applying a form of topic modeling to the Wikipedia English corpus).
ments are conceived of as "realizations" of these topics, i.e., topic 1 is associated with a document with 5% probability, topic 2 with 10% probability, and so forth.\textsuperscript{183}

Traditional topic modeling provides a method that enables researchers to discover textual "structure," but it is somewhat incomplete in the context in which we are working because we are interested not only in those terms that are most likely to belong to the same underlying idea, but also in those terms that have the strongest association with the outcome, as reflected by the MNIR factor loading coefficient above. In terms of Figure 1 in subpart III.A, we are interested not only in the vertical dimension (how tightly phrases co-occur together and are likely to reflect the same underlying idea) but also in the horizontal dimension (how well phrases predict veil-piercing outcomes). If we were to merely order the words within topics by the strength of the topic-word association, we would lose important information, namely, each phrase's correlation with the judicial outcome.

Accordingly, in addition to applying topic modeling to the two-word phrases in our dataset, for purposes of interpretation we further "weigh" the topic-phrase probabilities by multiplying each by the phrase's MNIR factor loading coefficient. This adjusts the topic-phrase probability by the strength of each phrase's association with the outcome. These adjusted probabilities are not used in the topic modeling estimation, but are employed solely to interpret the results in light of the predictive power of the underlying phrases.

As Table 5 shows, applying topic modeling to the dataset yields a remarkable result: despite several meaningless "noise" terms, a three-topic model yields terms that correspond rather precisely to the three justifications for piercing that we articulate in this Article. We have listed the top twenty-five terms for each topic below and bolded the terms particularly relevant to each theory.

\textsuperscript{183} For an algebraic presentation, see the Appendix to this Article.
Table 5. Topic Modeling—Three Topics

<table>
<thead>
<tr>
<th>Topic 1: Statutory Scheme</th>
<th>Topic 2: Misrepresentation</th>
<th>Topic 3: Bankruptcy Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>worker[s'] compens[ation]</td>
<td>breach contract</td>
<td>fraudul[ent] transfer</td>
</tr>
<tr>
<td>compens[ation] act</td>
<td>term contract</td>
<td>reason equival[ant]</td>
</tr>
<tr>
<td>sherman act</td>
<td>good faith</td>
<td>substant[ive] consolid[ation]</td>
</tr>
<tr>
<td>pension fund</td>
<td>neglig[ent] misrepresent[ation]</td>
<td>transfer made</td>
</tr>
<tr>
<td>statement undisput[ed]</td>
<td>enter contract</td>
<td>prop[erty] estat[e]</td>
</tr>
<tr>
<td>independ[ent] contractor</td>
<td>limit partnership</td>
<td>hinder delay</td>
</tr>
<tr>
<td>compens[ation] benefit[s]</td>
<td>commerci[al] code</td>
<td>breach fiduciary[y]</td>
</tr>
<tr>
<td>pension plan</td>
<td>franchis[e] agreement</td>
<td>bankrupt[y] estat[e]</td>
</tr>
<tr>
<td>trade secret</td>
<td>sale contract</td>
<td>actual intent</td>
</tr>
<tr>
<td>virgin island</td>
<td>contract sale</td>
<td>intent hinder</td>
</tr>
<tr>
<td>releases hazard[ous]</td>
<td>uniform commerci[al]</td>
<td>delay defraud</td>
</tr>
<tr>
<td>dispos[e] hazard[ous]</td>
<td>term agreement</td>
<td>avoid transfer</td>
</tr>
<tr>
<td>lanham act</td>
<td>breach warrant[y]</td>
<td>bankrupt[y] trust[e]</td>
</tr>
<tr>
<td>public util[ity]</td>
<td>perform contract</td>
<td>confirm plan</td>
</tr>
<tr>
<td>consent order</td>
<td>express warrant[y]</td>
<td>uniform fraudul[ent]</td>
</tr>
<tr>
<td>unfair competit[ion]</td>
<td>perpetrate fraud</td>
<td>ponzi scheme</td>
</tr>
<tr>
<td>cours[e] employ[ment]</td>
<td>assign leas[e]</td>
<td>file bankruptc[y]</td>
</tr>
<tr>
<td>benefit plan</td>
<td>partnership agreement</td>
<td>debtor insolv[ent]</td>
</tr>
</tbody>
</table>

Out of the three justifications, upholding a statutory scheme and promoting bankruptcy values have particularly strong, clear representation in the topic model. The second theory is less clear, likely because the precise facts of misrepresentation cases can vary substantially, leaving only terms related to a common topic (i.e., contract law) and key terms reflecting the misrepresentation such as “perpetr[ating] [a] fraud” on creditors. We have included the results of estimating four-, five-, and six-topic models in the online appendix to this Article, which similarly show that topics organize more or less along the lines of the three-theory justification we posit, with additional topics generally reflecting more nuanced distinctions between specific types of statutory regimes.184

184 See Macey & Mitts, supra note 181, at *127. For example, topic 1 in the five-topic model focuses on the income tax statute, whereas topic 4 reflects workers compensation and ERISA statutes.
These findings demonstrate that the ideas that co-occur within the text of the veil-piercing judicial opinions reflect the three-justification theory we articulate in this Article. Moreover, undercapitalization does not emerge as a leading set of phrases in any of these topics. While we expected this result in light of the very small MNIR coefficients for the undercapitalization phrases shown in the prior section, it is remarkable that the most natural grouping of ideas in the text of veil-piercing decisions that have predictive power track the theories we advance in this Article.

CONCLUSION

Scholars often claim that the inquiry into whether the corporate form should be disregarded has become oddly separated from the question of why the corporate form should be disregarded. In particular, the generic justifications such as undercapitalization, failure to observe corporate formalities, and preventing injustice offered to justify piercing are unpersuasive because they are either complete non sequiturs or vacuous due to the fact that they are wholly conclusory.

In this Article we seek to construct a rational framework for conceptualizing the circumstances in which it is appropriate and consistent with sound public policy to pierce the corporate veil. We do this by identifying the three public policy goals that explain the reasons why courts actually pierce the corporate veil. Our hypothesis is that the corporate veil will be pierced if, and only if, doing so is required for any one of the following three reasons: (1) to achieve consistency and compliance with the goals of a clear and specific extant regulatory or statutory scheme such as environmental law or unemployment law; (2) when there is evidence of fraud or misrepresentation by companies or individuals trying to obtain credit (and particularly where such misrepresentations lead a creditor erroneously to think that an individual shareholder of a company is guaranteeing what ostensibly is corporate indebtedness); and (3) when respecting the corporate form facilitates or enables favoritism among claimants to the cash flows of a firm and thus is inconsistent with the well-established bankruptcy law value of achieving the resolution of a bankrupt's estate that conforms both to contract law principles and to the priorities among claimants established by state law.

In addition to having greater explanatory power than current theories, our framework offers additional advantages over existing accounts of veil piercing. First, our three-part taxonomy distinguishes between situations in which courts actually pierce the corporate veil and situations in which the court does not ignore the legal form of a business organization but decides that certain liabilities that ostensibly are the corporation's alone actually should be shared by others in-
cluding shareholders and affiliate corporations. These cases, while they often are described as piercing cases, actually are more accurately characterized as joint and several liability cases. In particular, where a court finds liability on the basis that such liability is necessary to avoid fraud or to prevent injustice, the judge is really saying that creditors who have been hoodwinked into thinking that certain shareholders or affiliate entities are serving as guarantors should be able to collect from such guarantors. Such a finding does not mean that the corporation is a fiction or that it does not exist. Rather it means simply that other entities besides the corporation are responsible for some or all of the company's indebtedness.

Current accounts of the rationales for disregarding the corporate form, particularly accounts that consider undercapitalization as a grounds for veil piercing, fail to reconcile the basic tension between the well-settled doctrine that corporations can be established for the very purpose of avoiding personal liability and the doctrine that the veil can be pierced to avoid injustice. Under our account, corporations can be established for the sole purpose of avoiding personal or corporate liability on the part of investors, but not when doing so is inconsistent with the goals of another regulatory or statutory scheme; when there is evidence of fraud or misrepresentation by companies or individuals trying to obtain credit; or when respecting the corporate form facilitates or enables favoritism among claimants to the cash flows of a firm.

Our account is, in essence, a taxonomy of justifications for piercing the corporate veil that is consistent with the venerable adage that corporations legitimately can be established for the sole purpose of avoiding personal or corporate liability on the part of investors. This adage is deeply inconsistent with the notion that shareholders can be held personally liable for the obligations of corporations that courts deem to be "undercapitalized," but it is not inconsistent with our account.

Finally, because our taxonomy provides specific and logical justifications for veil piercing, it stands as a refutation of the assertion made by other scholars that the doctrine of piercing the corporate veil ought to be jettisoned. Rather than abandoning the practice of piercing the corporate veil, the most efficient approach to this doctrine would be for judges to determine whether piercing the corporate veil in a particular case will achieve one of the three public policy goals that we identify in this Article.
In this Appendix, we summarize the multinomial inverse regression (MNIR) model and provide an algebraic overview of the topic model we utilize in the body of the Article.\(^{185}\) MNIR utilizes "the inverse conditional distribution for text given \(y_i\) . . . to obtain low-dimensional document scores that preserve information relevant to \(y_i\),"\(^{186}\) where \(i\) represents each document in the dataset. It can be shown that under certain weak assumptions, the inverse distribution of \(x_i\) given \(y_i\) permits obtaining a sufficient reduction score \(z_i\), which renders \(y_i\) conditionally independent of \(x_i\) given \(z_i\).\(^{187}\) These sufficient reduction scores are obtained through a factor loading \(q_j\) for each of the \(j\) phrases in the vocabulary. In algebraic terms, the model is given by:

\[
x_i = \sum_{j=1}^{p} q_j x_{ij} \quad \text{for each } i \in \mathcal{Y}
\]

\[
x_i \sim \text{MN}(q_i, m_i) \quad \text{with } q_j = \frac{\exp[a_j + y_i]}{\sum_{j=1}^{p} \exp[a_j + y_i]} \quad \text{for } j = 1, \ldots, p, \ y_i \in \mathcal{Y}
\]

where each MN is a \(p\)-dimensional multinomial distribution with size \(m_i = \sum_{j=1}^{p} x_{ij}\) and probabilities \(q_i = [q_{i1}, \ldots, q_{ip}]\)' that are a linear function of \(y\) through a logistic link. . . . [T]he sufficient reduction (SR) score for \(f_i = x_i/m_i\) is then \(z_i = \Phi^T f_i \Rightarrow y_i \perp x_i, m_i \mid z_i.\)

The algebraic representation of our topic model is:

With each observation \(x_i \in \{x_1, \ldots, x_n\}\) a vector of counts in \(p\) categories, given total count \(m_i = \sum_{j=1}^{p} x_{ij}\), the \(K\)-topic model has \(x_i \sim \text{MN}(\omega_1 \theta_1 + \cdots + \omega_k \theta_K, m_i)\) where topics \(\theta_K = [\theta_{k1}, \ldots, \theta_{kp}]\)' and weights \(\omega_i\) are probability vectors. . . . In this context, each \(x_i\) is a vector of counts for terms (words or phrases) in a document with total term-count \(m_i\), and each topic \(\theta_k\) is a vector of probabilities over words. Documents are thus characterized through a mixed-membership weighting of topic factors and, with \(K\) far smaller than \(p\), each \(\omega_i\) is a reduced dimension summary for \(x_i.\)

Document-topic probabilities are estimated by the empirical frequency of words/phrases in each document and topic-word/-phrase probabilities are estimated by the co-occurrence of words across documents. The result of topic modeling is a "natural" probabilistic mapping of the documents to a small number of topics and a similar probabilistic mapping of words/phrases to documents. The choice of


\(^{187}\) Taddy, *supra* note 185, at 755.

\(^{188}\) Id.

which words and phrases are associated with which topic, and which topics are associated with which documents, is made by a method similar to traditional maximum likelihood estimation: which associations are most "likely" given the observed data.\textsuperscript{190}

\textsuperscript{190} More technically, we estimate $\Theta$ and $\Omega$ using joint maximum a posteriori (MAP) estimation under the natural exponent family parameterization. \textit{See id.} at 1186.